Political Competition in Legislative Elections*

Stefan Krasa† Mattias Polborn‡

March 10, 2014

Abstract

We develop a theory of candidate nomination processes predicated upon the notion that members of the majority party in a legislature collaboratively influence policy. Because of this team aspect, a candidate's party label matters for voters, in addition to his own policy positions: For example, in a liberal district, electing even a liberal Republican may be unattractive for voters because it increases the chance that Republicans obtain the majority in Congress, thereby increasing the power of more conservative Republicans. We show that candidates may be unable to escape the burden of their party association, and that primary voters in both parties are likely to nominate extremist candidates. We also show that gerrymandering affects the equilibrium platforms not only in those districts that become more extreme, but also in those that ideologically do not change.

Keywords: Differentiated candidates, primaries, polarization.

*Both authors gratefully acknowledge financial support from National Science Foundation Grant SES-1261016. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the National Science Foundation or any other organization. We are thankful for helpful comments by seminar participants at Illinois, Princeton, Western Illinois, Iowa, Michigan, Notre Dame and UC Irvine.

†Department of Economics, University of Illinois, 1407 W. Gregory Dr., Urbana, IL, 61801. E-mail: skrasa@uiuc.edu

‡Department of Economics and Department of Political Science, University of Illinois, 1407 W. Gregory Dr., Urbana, IL, 61801. E-mail: polborn@uiuc.edu.

1 Introduction

In the most basic model of representative democracy, voters elect legislative representatives whose positions reflect the preferences of their respective districts' median voters. These representatives convene in an amorphous assembly (one in which there are no parties, or parties at least do not play an important role), and national policy is set, in equilibrium, to correspond to the preferences of the median representative in this assembly. Thus, in this basic model, the legislature is composed of representatives who are very moderate relative to the voters who elect them, and actual policy and legislation reflects the most moderate position in this assembly of moderates. Suffice it to say that few observers of Congress believe that reality corresponds closely to these predictions; the central question is why this is the case.

In this paper, we build a model of electoral competition that can account for a much higher degree of polarization in the legislature, and which is based on two realistic ingredients: First, the majority party in a legislature is an important power center influencing the crafting of policy. Coordination of decision-making and voting according to the majority preferences in the majority party increases the influence of each majority party legislator on the policy outcome (Eguia, 2011a,b). Second, legislative candidates are nominated by policy-motivated primary voters who take both the general election and the legislation process into account when deciding whom to nominate.

The importance of parties is uncontroversial among scholars of legislatures. However, there is surprisingly little analysis of how the fact that each candidate is connected to a party and thus, implicitly, to the positions of candidates of that party from other districts influences the types of candidates who are nominated by their party to run for legislative office and the outcomes of elections in different legislative districts.

If one were to apply the simplest Downsian model naively to Congressional elections – which much of the empirical literature implicitly does – then it generates counterfactual predictions: In each district, both candidates should adopt the preferred position of the district median voter, and so, policy-wise, all voters should be indifferent between the Democratic candidate and his Republican opponent. Republicans in New England or Democrats in rural Southern districts should have a substantial chance to be elected to Congress if only they match their opponent's policy platform. Furthermore, in this model framework, gerrymandering districts would not help parties, at least not in the sense that it would increase the party's expected representation in Congress. It is safe to say that both of these predictions are counterfactual, but understanding why that is so is challenging.

In our model, the implemented policy is determined by a function that maps the ideal positions of majority party legislators into a policy, and satisfies some basic intuitive requirements such as efficiency and monotonicity. In the general election for the legislature, voters vote for their preferred candidate, taking into account the two ways in which their local representatives may change the policy outcome: First, the district result may change which party is the majority party in Congress, and second, if they elect a candidate who will be in the majority party, they may affect the ideological composition of the majority party.

In this framework, there are spillovers between different districts: The electoral prospects of candidates in a given district are influenced by the expected ideological position of their parties' winning candidates elsewhere. The association with a party that is not attuned with a district's ideological leanings may be poisonous for a candidate even if his own policy positions are tailor-made for his district.

Consider, for example, Lincoln Chafee, the U.S. senator from Rhode Island from 1999 to 2006. In spite of being a Republican, Chafee had taken a number of moderate and liberal positions that brought him in line with voters in his state.[1] In the 2006 election, "exit polls gave Senator Lincoln Chafee, a popular moderate Republican from a long-admired political family, a 62 percent approval rating. But before they exited the polls, most voters rejected him, many feeling it was more important to give the Democrats a chance at controlling the Senate. [...] 'I'm caught between the state party, which Im very comfortable in, and the national party, which I'm not,' said Mr. Chafee."[2] His Democratic challenger Whitehouse "succeeded by attacking the instances in which Chafee supported his party's conservative congressional leadership (whose personalities and policies were very unpopular, state-wide)."[3]

In a review of 2006 campaign ads, factcheck.org summarized: "President Bush was far and away the most frequent supporting actor in Democratic ads [...] The strategy is clear: whether they're referring to a Republican candidate as a 'supporter' of the 'Bush agenda' or as a 'rubberstamp,' Democrats believe the President's low approval ratings are a stone they can use to sink their opponents [...] Democratic Sen. Hillary Clinton of New York got the most mentions in Republican ads holding forth the supposed horrors of a Democratic-controlled Senate [...] The runner-up is 'San Francisco Liberal Nancy Pelosi,' who is mentioned in at least 6 GOP ads as a reason not to vote for a Democrat who would in turn vote to make her Speaker of the House."[4]

[1]For example, Chafee was pro-choice, anti-death-penalty, supported gay marriage and voted against the Iraq war (see http://en.wikipedia.org/wiki/Lincoln_Chafee).

[2]"A GOP Breed loses its place in New England", New York Times, November 27, 2006.

[3]See http://en.wikipedia.org/wiki/Lincoln_Chafee.

[4]See http://www.factcheck.org/elections-2006/our_2006_awards.html

We show that "contamination" – as we call this spillover effect – makes most legislative elections uncompetitive and results in an equilibrium in which party members are able to nominate their ideal candidate, rather than the ideal candidate of the district median voter, and nevertheless win by a healthy margin. The other party either cannot effectively compete because, even if it nominates a candidate at the ideal position of the median district voter, that voter still prefers the more extreme competitor because he is associated with an average party position that is ideologically preferred by the district median voter; or the other party could, in principle, compete, but prefers to nominate a losing extremist. The latter case arises if a winning moderate might "taint" the party's position in the legislature.

Again, Lincoln Chafee provides an instructive illustration of this principle. Before the 2006 general elections, when Republicans had a clear majority in the Senate, conservative Republicans in Rhode Island mounted a primary challenge. Chafee defeated his challenger who had attacked him for not being sufficiently conservative only by a margin of 53 percent to 47 percent, and there is reason to believe that a majority of "real" Republicans would have preferred to replace a popular incumbent Senator[5] with an extremist whose policy positions would have implied a very low likelihood of prevailing against the Democrat in the general election in Rhode Island. Our model explains why this behavior may be perfectly rational for policy-motivated Republicans: From their point of view, having Chafee as a member of the Republican Senate caucus caused more harm than good.

In contrast to the classical one-district spatial model, the ideological composition of districts in our model does not only influence the ideological position of elected candidates, but also the chances of parties to win. Thus, partisan incentives for gerrymandering are much larger in our model. We also show that gerrymandering or, more generally, the intensification of the median ideological preferences in some districts, also affects the political equilibrium in those districts where the median voter preferences remain moderate. Thus, our results imply that testing for the causal effect of gerrymandering on polarization in Congress is more complicated than the existing literature has recognized.

Our paper proceeds as follows. Section 2 reviews the related literature. In Section 3, we provide some stylized facts about statewide executive and legislative elections, and explain why they are hard to explain within the standard model that looks at legislative elections in different districts in isolation. Section 4

[5]Rhode Island's open primary system allows registered Democrats and Independents to vote in the Republican primary. The New York Times article "To hold Senate, GOP bolsters its most liberal" (September 10, 2006) quotes a Republican consultant as saying that "Theres no doubt that if the primary was held only among Republicans, Chafee would lose. He would be repudiated by the Republicans who he has constantly repudiated."

presents a simplified example. In Section 5, we set up the general model, and the main analysis follows in Sections 6 and 7. We conclude in Section 8.

2 Related literature

Ever since the seminal work of Downs (1957), the position choice of candidates and the determinants of policy convergence or divergence are arguably the central topics in political economy models of elections. While the classical median voter framework identifies reasons for equilibrium platform convergence, there is a large number of subsequent variations of the spatial model of electoral competition that develop different reasons for policy divergence, such as policy motivation (e.g., Wittman 1983; Calvert 1985; Martinelli 2001; Gul and Pesendorfer 2009); entry deterrence (e.g., Palfrey 1984; Callander 2005); and incomplete information among voters or candidates (e.g. Castanheira 2003; Callander 2008; Bernhardt et al. 2009).

Overwhelmingly, the existing literature looks at isolated elections – usually, two candidates compete against each other, and voters care only about their positions. In the probabilistic voting model (e.g., Hinich 1978; Lindbeck and Weibull 1987; Dixit and Londregan 1995; Banks and Duggan 2005), voters also receive "ideological" payoffs that are independent of the candidates' positions. While, to the best of our knowledge, these authors do not interpret the ideological payoff as capturing the effects of the candidate being affiliated with a party, and therefore implicitly the party's other legislators' policy positions, this is a possible interpretation. However, the "ideology shock" in these models is exogenous, so that the main point of interest in our model – How does the fact that policy is determined within the legislature, rather than unilaterally by the local candidate, affect both the candidates' equilibrium positions and the voters' choice between local candidates? – cannot be analyzed in these models.

Our model belongs to the class of differentiated candidates models (Soubeyran 2009; Krasa and Polborn 2010a,b, 2012, 2013; Camara 2012). In these models, candidates have some fixed "characteristics" and choose "positions" in order to maximize their probability of winning. Voters care about outcomes derived from a combination of characteristics and positions. In contrast to existing differentiated candidates models, voters' preferences over characteristics (i.e., the candidates' party affiliations) are endogenously derived from the positions of Democrats and Republicans in other districts.

Erikson and Romero (1990) and Adams and Merrill (2003) introduce an influential model framework in the political science literature in which voters receive, in addition to the payoff from the elected candidate's position, a "partisan" payoff from the candidate's party affiliation. However, this partisan payoff is not derived from any multidistrict model, and is in fact orthogonal to the policy positions chosen by the candidates. The contribution of our model to this literature is to show that one can interpret it as providing a microfoundation for these partisan payoffs: The association of the local candidates with the two parties matters because the parties in the legislature determine the implemented policy, and so the fact that voters care about the candidates' party labels is perfectly rational in our model, and the degree to which it matters for voters depends on the equilibrium polarization between the parties' candidates in other districts.

The legislative part of our model assumes that parties in Congress have a strong influence on policy outcomes. A significant number of models explain why parties matter. Conditional party government theory (Rohde, 2010; Aldrich, 1995) and endogenous party government theory (Volden and Bergman, 2006; Patty, 2008) argue that party leaders can use incentives and resources to ensure cohesiveness of their party. Procedural cartel theory (Cox and McCubbins, 2005) argues that party leadership can at least enforce voting discipline over procedural issues, and Diermeier and Vlaicu (2011) provide a theory where legislators endogenously choose procedures and institutions that lead to powerful parties. All these models of the importance of parties in Congress take the preference distribution of legislators as exogenously given, while our model provides for an electoral model and thus endogenizes the types of elected legislators.

Since we assume that the nomination decision is made by a policy-motivated party median voter, our model is related to the literature on policy-motivated candidates pioneered by Wittman (1983) and Calvert (1985), who assume that *candidates* are the ones who are policy-motivated and get to choose the platform that they run on. In our model, the effective choice of platform is made by the primary election median voter,[6] but this change does not substantively affect the analysis. This approach is also taken by Coleman (1972) and Owen and Grofman (2006). To our knowledge, no paper in this literature analyzes policy-motivated policy selectors in the type of "linked" elections in different districts that we focus on.

Our results are relevant for the large empirical literature that analyzes how primaries, the ideological composition of districts and especially the partisan gerrymandering of districts affects the ideological positions of representatives in Congress (e.g., Lee et al. 2004; McCarty et al. 2009; Hirano et al. 2010). Most empirical papers in this literature do not include a formal model from which they derive predictions about the "expected" correlations, but rather take the intuition from the isolated election model and simply transfer

[6] Implicitly, we assume that either candidates can commit to an ideological position in the primary, or that candidates are citizen-candidates with an ideal position that is common knowledge.

them to the setting of legislative elections. For example, there is a general expectation in the empirical literature that the positions of district representatives, i.e. U.S. Senators or House members, measured by their DW-Nominate score should more or less reflect the conservativeness of their districts. Our model shows that this transfer of results derived in the isolated-election model to legislative elections is not always justified, and that the candidates' equilibrium positions may correspond to the preferences of the parties' respective primary electorates rather than those of the district median voter.

3 Consistent lopsided elections: A puzzle for the single-district model

In this section, we argue that the influence of the electorate's preference distribution on the parties' performance is substantially larger in legislative elections than in executive ones. This stylized fact is puzzling when viewed through the lens of the simplistic one-district spatial model which does not distinguish between executive and legislative elections. As we show, one can interpret our model as a resolution of this puzzle.

3.1 Some stylized facts

The simplest Downsian model predicts that both candidates in a plurality rule election choose their position at the median voter's ideal point, so that all voters are indifferent between the candidates. A rather liberal or conservative district should not provide a particular advantage – in terms of the probability of winning the district – to Democrats or Republicans. In Section 3.2, we look at somewhat more sophisticated one-district models of candidate competition, but argue that this intuition is quite robust.

In practice, it is well known that the ideological preferences of voters do affect the electoral chances of the different parties' candidates – we talk of "deep red" (or blue) states, implying that the candidates of the ideologically favored party have a much clearer path to victory than their opposition.

However, we now argue that the voters' ideological preferences have a substantially larger effect in legislative elections than in executive ones. To demonstrate this phenomenon, we consider Gubernatorial and U.S. Senate elections from 1978 to 2012. Both of these types of contests are state-wide races, but evidently, Gubernatorial elections are for executive positions while Senate elections are for legislative ones. Consistent with the empirical literature, we measure the median state ideology by its Partisan Voting Index (PVI), which is calculated as the difference of the state's average Democratic and Republican Party's vote share in the past two U.S. Presidential elections, relative to the nation's average share of the same.[7]

The dependent variable is the difference between the Democrat's and the Republican's vote share of the two party vote in a particular election. In addition to the main independent variables of interest (PVI and $PVI\times$Senate election), we use incumbency dummies and year fixed effects in order to control for the electoral advantage of incumbents, and for election-cycle national shocks in favor of one party.

Table 1 summarizes the results, with the first column as the baseline case (all years since 1978, all states). For Gubernatorial elections (the omitted category), the PVI coefficient indicates that a one point increase in the Democratic vote share in Presidential elections increases the Democratic gubernatorial candidate's vote share only by about 0.519 points. In contrast, in Senate elections, the same ideological shift increases the Democratic Senate candidate's vote share by $0.519 + 0.645 = 1.164$ points, more than twice the effect in Gubernatorial elections; evidently, the difference between executive and legislative elections is substantial and highly significant. The remaining three columns confirm the qualitative robustness of this difference if we restrict to elections after 1990 and if we exclude the political South.[8]

Table 1: Senate and Gubernatorial elections

	All States		Without Confederacy States	
	1978-2012	1990-2012	1978-2012	1990-2012
PVI	0.519***	0.589***	0.529***	0.614***
	(0.111)	(0.124)	(0.117)	(0.132)
PVI× Senate	0.645***	0.596***	0.597***	0.514***
	(0.149)	(0.167)	(0.156)	(0.177)
N	1103	702	871	553
R^2	0.551	0.595	0.571	0.62

*** indicates significance at the 1% level.
Additional explanatory variables used: Election type (Senate or Governor), year dummies, and incumbency status.
Data Source: Congressional Quarterly, http://library.cqpress.com/elections/

A coefficient of about 1 for Senate elections is quite remarkable — if Senate candidates were hard-wired

[7]For example, if, in a particular state, Democratic presidential candidates run ahead of Republicans by 7 percent (on average in the last two elections), while nationally, Democratic candidates win by 3 percent (in the same two elections), then the state has a PVI of $7\% - 3\% = 0.04$. Also note that vote shares are calculated relative to the two-party vote, i.e., votes for minor parties are eliminated before the vote share percentages are calculated. See http://cookpolitical.com/application/writable/uploads/2012_PVI_by_District.pdf for the PVI based on the 2004 and 2008 Presidential elections.

[8]The reason for excluding the South is that, at least until the 1990s, there were a lot of conservative Southern Democrats in state politics in the South, so it is useful to check that our results are not just driven by this region of the country.

at their Presidential party position, irrespective of whether such a position is competitive in their respective state, then this should result in a coefficient of (about) 1. Any degree of willingness of the disadvantaged candidate to adjust his position to better fit the state's voter preferences should reduce the advantage of the opponent, and thus the estimated coefficient. Somehow, only gubernatorial candidates appear (at least to some extent) capable of such a position adjustment, while Senate candidates are not.

3.2 Inconsistency with the simple single-district model

These stylized facts are difficult to reconcile with the standard model of political competition that implicitly assumes that the electoral competition between the two candidates in each district is not influenced by what happens outside the district. Specifically, it is very difficult to set up a one-district model in which a particular party wins almost certainly, and does so with a substantial winning margin.

Without loss of generality, let the median voter be located at zero and the party medians at m_D and m_R. Even if party medians are far apart from each other, parties have to nominate relatively moderate candidates in order to remain competitive. This is obvious for the model without uncertainty where both parties nominate candidates that maximize the median voter's utility, i.e., $x_D = x_R = 0$, and both parties have equal vote shares, even if one party's ideal point is substantially closer to the median voter's ideal position than the opposition's. Since this is true for arbitrary ideal positions of the parties, it implies that, even if party members become more extreme, i.e., m_D moves to the left and m_R to the right, the equilibrium policies remain moderate, and the margin of victory is close to zero (if the distribution of voter types is continuous). Statement 2 of Proposition 2 in Section 7 below shows that this insight also extends to the case with uncertainty about the median. In particular, even if the positions of the primary voters $-m$ and m go to $-\infty$ and $+\infty$, the position of the candidates remain moderate (i.e., bounded).

Thus, if party members become more extreme then the Downsian model predicts at most a small effect on policy: Party members continue to nominate moderate candidates, and both parties receive approximately one-half of the votes. In contrast, it is a widespread view that the rise of activist and more ideological party members has resulted in more extreme candidates being nominated for office (e.g., Fiorina et al. (2006)). Further, many political commentators and scholars diagnosed a rise in polarization between the two parties. In order to generate such polarization in a standard model with policy-motivation, uncertainty, e.g., about the median voter's location, must increase. In other words, we would need that the quality of political polls deteriorates over time, which is somewhat implausible.

The prediction that both candidates *in executive elections* (i.e., those where the elected candidate can set policy without being tied to their party) will be competitive is borne out in U.S. presidential elections. For example, between 1988 and 2012 the difference between the Republican and Democratic vote share in Presidential elections was between -5.6% and 7.7%, with a median of -0.5%.[9] Furthermore, the results of Table 1 above indicate that Gubernatorial elections, even in ideologically skewed states, are at least considerably more competitive than the Presidential election in those same districts. In contrast, as shown above, many legislative elections result in one party receiving a substantially higher vote share than its opposition.

Can we generate lopsided outcomes if one of the candidates has a "valence" advantage? Suppose that the net-valence of the Republican candidate, $v_R - v_D$ is $\varepsilon > 0$. Then, in equilibrium, $x_D = 0$ and $x_R = \sqrt{\varepsilon}$. Given these positions, the median voter at 0 is again indifferent: If he votes for D the utility is 0, if he votes for R the utility is $-x_R^2 + \varepsilon = 0$, but in equilibrium he supports the Republican who wins the election. If voter types are continuously distributed, then the margin of victory is (almost) zero.

In order to generate a vote margin that is bounded away from 0, one would have to assume a valence advantage that is so large that the median voter prefers the favored candidate even if he is located at his party median's ideal point, and the opposition candidate is located at the median voter's ideal point. Usually, valence is interpreted as a small personal preference; for rational voters, the most of the utility-relevant payoff from a legislature should come from the laws the legislature enacts, rather than from legislators' valences. Furthermore, it would be hard to understand why one party should be consistently much better than the other party in terms of the quality of candidates that they select, and why that party should necessarily be the one that is ideologically closer to the median voter.

It is easy to show that, if there is some uncertainty about the median, then the higher valence candidate wins with probability close to one, but the margin of victory is close to zero. However, if the uncertainty is not too large, then the winning margin is close to zero. The reason is that in any Downsian model without uncertainty, the median voter is indifferent between the candidates, while with some uncertainty he is close to indifferent, and hence the electorate splits close to 50-50. One of the key insights of this paper is that this is not longer true in a multi-district setting.

[9]U.S. presidents are elected in many districts through the electoral college system rather than by a majority of the popular vote. However, to the extent that state ideological leanings are fixed and known, the objective for the parties' primary electorates is essentially to nominate a candidate who can win in the decisive swing state.

4 A simple example

Before turning to our general model, we present an example that, while based on a somewhat simplified utility function, can illustrate some interesting effects. We change the assumption of the "naive standard model" that voters care *only* about the positions of their local candidates and do not regard the positions of fellow party members with whom their representative would caucus if elected. In contrast, suppose now that voters understand that policy is not set by their representative alone, but rather that the positions of other representatives have a significant influence.

Specifically, let $x_{i,D}$ and $x_{i,R}$ be the positions of the local candidates for Congress, and ξ_D and ξ_R the expected median Democratic and Republican positions in Congress (the voters' expectations are correct in equilibrium). Assume that a voter with ideal position θ who votes for the Democrat has utility

$$-\gamma(\theta - x_{i,D})^2 - (1-\gamma)(\theta - \xi_D)^2, \tag{1}$$

and a utility of voting for the Republican equal to

$$-\gamma(\theta - x_{i,R})^2 - (1-\gamma)(\theta - \xi_R)^2. \tag{2}$$

Consider the case of a continuum of districts, so we can disregard here any influence that the local Democratic or Republican candidate would have on the corresponding Congressional caucus, if elected (in the general model, we will also take this effect into account).

Primary voters have ideal positions of -1 (Democrats) or $+1$ (Republicans) in all districts, and aim to nominate candidates who are as close as possible to their own ideal position, subject to the constraint that they can win in the general election. If no candidate exists who can win against the equilibrium candidate of the other party, then primary voters nominate the most competitive candidate (they do this, in equilibrium, in order to limit the extremism of the other party's candidate).

There is a continuum of districts whose median voters are uniformly distributed between $-\mu$ and μ, where $\mu < 1$ (i.e., in all districts, general election median voters are less extreme than primary voters).

We look for a symmetric equilibrium in which conservative districts vote for Republicans and liberal ones for Democrats (one can show that this is the only type of equilibrium). Consider a conservative district i with a median voter $M_i > 0$. The maximally conservative candidate who still wins in this district makes the median voter indifferent to the Democratic candidate (who, as assumed above, is located at the most competitive position M_i). Thus

$$-\gamma(M_i - x_{i,R})^2 - (1-\gamma)(M_i - \xi_R)^2 = -(1-\gamma)(M_i - \xi_D)^2 - \gamma(M_i - M_i)^2. \tag{3}$$

Using symmetry (i.e., $\xi_D = -\xi_R$) and solving for $x_{i,R}$ gives

$$x_{i,R} = M_i + 2\sqrt{\frac{1-\gamma}{\gamma} M_i \xi_R}. \tag{4}$$

The right-hand side of (4) consists of two terms: The first one is the median voter's ideal position, and the second one is the leeway that the Republican primary voter has to choose a more conservative candidate because the median voter in a district $M_i > 0$ prefers the position of other Republicans, ξ_R to that of the other Democrats, ξ_D. This leeway arises because, even though the Democratic candidate in district i proposes M_i's ideal position, he is *contaminated* by his association with the Democratic party whose representatives are on average too liberal for the taste of the conservative median voter in district i. The extent of this leeway depends on the relative importance of this effect as captured by $1-\gamma$, and on the extent of the median voter's preference for the generic Republican position, which depends on the median voter's ideal position M_i.

Of course, if the right-hand side of (4) is larger than 1, the primary election median voter simply nominates his ideal candidate $x_{i,R} = 1$, and the median voter has a *strict* preference for the Republican candidate, so that this candidate receives a strict supermajority of votes.

The position of the median Republican legislator in Congress, ξ_R, is determined endogenously in equilibrium. Since (4) is evidently increasing in M_i, and since Republicans win exactly the districts in $[0, \mu]$, it is equal to the position of the Republican legislator elected from district $\mu/2$. Substituting, we get

$$\xi_R = \begin{cases} \frac{\mu}{2} + 2\sqrt{\frac{1-\gamma}{\gamma}\frac{\mu}{2}}\sqrt{\xi_R} & \text{if } \frac{\mu}{2} + 2\sqrt{\frac{1-\gamma}{\gamma}\frac{\mu}{2}} \leq 1 \\ 1 & \text{otherwise} \end{cases} \tag{5}$$

Solving for an interior solution (i.e., in the first line of (5)), we get

$$\xi_R = \frac{\mu}{2}\left[\frac{1 + \sqrt{1-\gamma}}{\sqrt{\gamma}}\right]^2 \tag{6}$$

For the case of $\gamma = 1$, i.e., if voters care exclusively about their local candidates and there is no spillover from party positions, we simply get that each elected candidate is located at the median voter's ideal position, and so the median Republican in Congress is just the median voter of the median conservative district, $\mu/2$, and the median Democrat in Congress is the median voter of the median liberal district, $-\mu/2$.

However, the smaller is γ, the larger is the factor in square brackets in (6). For example, if $\gamma = 1/2$, i.e. both the local candidates' positions and the national party positions weigh equally for voters' decisions, then the term in square brackets is $2\left(1 + \frac{1}{2}\sqrt{2}\right)^2 \approx 5.88$, generating a lot more polarization in Congress.

The equilibrium winning position in a given district depends on the extent of preference heterogeneity between districts. We can think of an increase in μ as a increased preference heterogeneity between districts, maybe brought about by gerrymandering that generates more "secure" Democratic and Republican districts. Holding constant the preferences of a particular moderate district (i.e., M_i), how does this change *in other districts* affect the equilibrium position of the winning candidate? If we have an interior value of ξ_R and substitute in (4), we get

$$x_{i,R} = M_i + \sqrt{2\mu}\frac{1 - \gamma + \sqrt{1 - \gamma}}{\gamma} \tag{7}$$

Thus, an increase in the heterogeneity between district median voters leads to more extreme positions of winning candidates even in those districts where the voter preference distribution remains unchanged.

This finding has important implications for the empirical analysis of the effects of gerrymandering. For example, McCarty et al. (2009) argue that, while Congress has become more polarized in a time during which electoral districts became more heterogeneous thanks to gerrymandering, this is merely a temporal coincidence. They draw this conclusion by arguing that also legislators from districts that were not gerrymandered (e.g., in the Senate, or in House districts in small states where there is less scope for gerrymandering) became more partisan in their voting behavior. Our example shows that this argument may be flawed because the candidates in districts that are not directly affected by gerrymandering (in the sense that their median voter's ideal position remains constant) nevertheless choose more extreme positions *because of* the evolution of more extreme districts elsewhere. We will return to this argument in Section 7.

5 Model

The determination of policy in the legislature. We consider a polity divided into $2n+1$ districts, ordered according to the ideological preferences of their respective median voters, so that district $n+1$ is the median district. Each district i elects one representative to the legislature, who is characterized by a position x_i. All candidates are attached to one of two parties, called Democrats and Republicans.

Let $x = (x_{i,D}, x_{i,R})_{i=1,\dots,2n+1}$ be the positions of candidates in all districts, and let X be the set of all such positions. Let $k_i \in \{D, R\}$ denote the party of the winning candidate in district i, and $K = (k_i)_{i=1,\dots,2n+1}$. Then the policy selection function ξ maps the candidates' ideal positions and the election outcomes in districts into an implemented policy, i.e. $\xi\colon X \times K \to \mathbb{R}$. As a simple example, suppose that ξ selects the preferred policy of the majority party's median legislator. The intuition for our results can most easily be understood with this policy selection function, but Proposition 1 considers a more general class of policy selection functions.

Voter utility. The utility of a voter with ideal position θ from district i is

$$u_\theta(x, K, v_i) = -(1-\gamma)(\xi(x, K) - \theta)^2 - \gamma(x_i - \theta)^2, \tag{8}$$

where $\gamma \in (0, 1)$. Here, the first term is the utility from the legislature's policy, and the second term is the utility from the policy position of district i's representative. Note that $\gamma \to 1$ corresponds to the standard case where voters only care about the election outcome in their own district, and $\gamma \to 0$ means that voters care only about the implemented policy and not their own representative's position per se.

Ex-ante there is uncertainty about the state of the world, captured by a collection $(\omega_i)_{i=1,\dots,2n-1}$ of random variables, one for each district, where $\omega_i \in \Omega_i$ captures the uncertainty pertaining to the ideal position of district i's median voter.

Timeline. The game proceeds as follows:

Stage 1 In each district, the local members of each party simultaneously select their candidates, who are then committed to their policies $x_{i,D}, x_{i,R} \in \mathbb{R}$. We assume that the nomination process can be summarized by the preference parameter of a "decisive voter," whom we take to be the median party member in the district, and whose ideal position is denoted m for Republicans and $-m$ for Democrats.

Stage 2 In each district i, the median voter $M_i(\omega_i)$ is realized, observes the candidate positions $x_{i,D}$ and $x_{i,R}$ in his own district, and chooses whom to vote for. Note that, for the other districts, the median voter does not observe the candidates' positions, but he has correct expectations in equilibrium.

Stage 3 The elected candidates from all districts form the legislature, which determines the implemented policy via function ξ, and payoffs are realized; no strategic decisions take place in this stage.

Equilibrium concept. We consider Perfect Bayesian Nash equilibria of this game. For completeness we now describe in detail the strategy spaces for the players. In a slight abuse of notation, let $k_i : (M_i(\omega_i), \omega_i, x_{i,D}, x_{i,R}) \to \{D, R\}$ denote the voting strategy of district i's median voter $M_i(\omega_i)$ when choosing between candidates $x_{i,D}$ and $x_{i,R}$ in state ω_i.

Definition 1 *A collection of policies $x \in X$ and of voter strategies $k(x, \omega) = (k_i)_{i \in \{1,\ldots,2n+1\}}$ is a pure strategy equilibrium if and only if*

1. *for every district i and for every state ω_i in that district, the median voter $M_i(\omega_i)$ chooses his optimal candidate: If $w_i(M_i(\omega_i), \omega_i, x_{i,D}, x_{i,R}) = P_i$, and $\overline{P}_i$ denotes the other candidate, then*

$$E_{\omega_{-i}} u_{M_i(\omega_i)}(x, k_i(\cdot), k_{-i}(\cdot), v_{i,P_i}(\omega_i)) \geq E_{\omega_{-i}} u_{M_i(\omega_i)}(x, k_i', k_{-i}(\cdot), v_{i,\bar{P}_i}(\omega_i)) \text{ for all } k_i' \in \{D, R\}$$

   where the expectation is taken over the realization of uncertainty in the other districts;

2. *the candidate choices of the decisive primary voters $m_{i,P} \in \{-m, m\}$ are optimal for them, respectively:*

$$E_\omega u_{m_{i,P}}(x, k_i(\cdot), v_{i,k_i}) \geq E_\omega u_{m_{i,P}}(x_{-i,P}, \tilde{x}_{i,P}, k_i(\cdot), v_{i,k_i(\cdot)}), \text{ for both parties } P \text{ and all alternative positions } \tilde{x}_{i,P}$$

   where the expectation is taken with respect to the ex-ante distribution of ω.

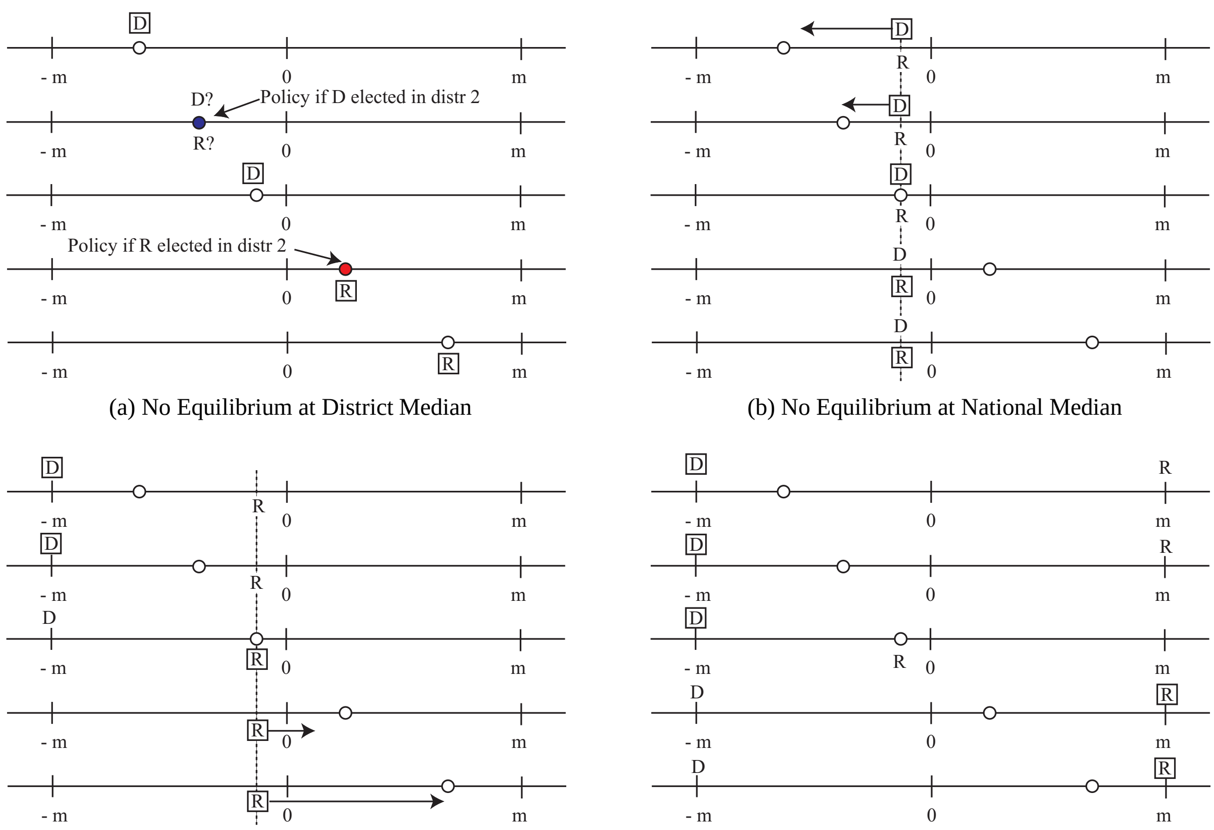
6 Equilibrium analysis in the case without uncertainty

6.1 An example of contamination

We start with a simple framework without uncertainty about the median voters' ideal points, because this already allows us to identify *contamination*, one of the key effects in our model. As a benchmark, remember that, if there is a single district with a median Democratic primary voter at $-m$, a Republican primary voter at m and a general election median voter located at M, then both primary voters in equilibrium select candidates with $x = M$. The general election median voter is indifferent between the parties, and may thus choose either the Democrat or the Republican with probability 1 in a pure strategy equilibrium.

To gain an intuition for the equilibrium in our model, consider the example in Figure 1: There are five districts, with general election median voters located at $M_1 < M_2 < \ldots < M_5$. Without loss of generality, suppose that $M_3 < 0$ (the case that $M_3 > 0$ is symmetric).

Suppose that the implemented policy is equal to the median of the majority party, and, for simplicity, consider only pure strategy equilibria. We first show that selecting candidates who are located at the median in each district is no longer an equilibrium, due to policy externalities between different districts.



(a) No Equilibrium at District Median

(b) No Equilibrium at National Median

(c) No Equilibrium if R is at National Median

(d) Equilibrium

Figure 1: Contamination with 5 districts

To see this, consider panel (a) of Figure 1 where the candidates in all districts are located at the ideal positions of their respective median voters, $x_{i,D} = x_{i,R} = M_i$. Assume that districts 1 and 3 elect their Democratic candidates, and districts 4 and 5 elect their Republican candidates (the small box around the party label indicates elected candidates' party and position). Consider the problem of district 2's median voter. If he elects the Democrat, the policy is equal to his preferred outcome, while if he elects the Republican, the national policy is M_4. Thus, he strictly prefers the Democrat.

However, if district 2's median voter strictly prefers the Democratic candidate, then the Democratic primary voter at $-m$ could select a more extreme candidate and still be guaranteed to win. Since this makes the Democratic primary voter strictly better off, the scenario depicted in Figure 1 (a) is not an equilibrium. Intuitively, in the one-district model, the general election median voter is indifferent when both parties choose the same policy, but with multiple districts this is no longer true since the positions of legislators from the other districts generate the externalities just discussed; and, if the median voter in a district is not indifferent, the party of the preferred candidate can nominate a more ideologically extreme candidate because that more extreme candidate will still be elected.

We now turn to Figure 1(b) which shows an equilibrium in which all parties nominate their ideal candidates, except for Republicans in district 3; they nominate the median voter's ideal candidate, but nevertheless lose to an extreme Democrat.

To see that this is an equilibrium, let us start by considering the most competitive district, district 3. While the *local* Republican candidate is ideal from the median voter's perspective, his election will move the national policy from $-m$ (the policy if the Democrat wins) to $+m$ (the median policy of the Republican legislators if they win a majority). Since the median voter in district 3 prefers $-m$ to m, he will stick to the Democrats even if they nominate the most extreme candidate, and so the Democratic primary voters can get away with nominating their ideal candidate.

Note that, in this equilibrium, voters split in each district according to which of the extreme positions offered by the parties ($-m$ or m) they prefer: All voter types with $\theta < 0$ vote for the Democrats, and all voter types $\theta > 0$ vote for the Republicans. Since there is no district in which the median is at 0, *all* districts have a winning margin that is bounded away from 0. Also, sufficiently small valence shocks, or uncertainty about the ideal position of the median voter, would therefore not change the qualitative features of the equilibrium, as sufficiently small uncertainty cannot change the outcome anywhere.

The equilibrium depicted in Figure 1(b) is not unique; for example, the moderate Republican who loses in district 3 could, of course, also locate at a more extreme position. Similarly, the extreme Republicans in districts 4 and 5 could locate at a more moderate position as long as it is not too moderate — it cannot be the case that district 3's general election median voter would prefer the position of the Republicans in districts 4 and 5 to the one of Democrats in districts 1 and 2, because then he would vote for the Republican; and once district 3 votes for the Republican candidate, the Republican primary voters in districts 4 and 5 would much rather nominate more extremist candidates.[10]

Why do Democrats in districts 4 and 5 (where they do not win) not nominate more moderate candidates? Indeed, more moderate Democrats would be elected in these districts because, as members of the majority party, they would moderate the implemented policy. However, that is exactly the reason why the Democrats do not want to nominate a more moderate candidate – if he were to be nominated and elected, the implemented policy would be less desirable for the decisive Democratic primary voter.

Finally, we should also note that the assumption that the general election median voters in the different districts have different ideal positions is decisive for the equilibrium polarization result. If all median voters are at the same position, i.e., $M_i = M$ for all i, then in equilibrium all candidates are located at M; of course, identical medians in all districts is a highly non-generic case.

6.2 The main result

We now show that the results of the example discussed in the previous section also hold for more general policy selection functions ξ. To do this, we formally define three key properties that ξ must satisfy.

1. The policy only depends on the set of ideal positions of majority party legislators.

2. The policy is Pareto efficient for the group of majority party members.

3. The policy is continuous and monotone in the ideal points of majority party legislators. For example, if the Democrats have the majority, and some Democratic legislators' ideal points shift to the left, then the resulting policy also shifts to the left.

In the remainder of this section, we formally define these properties and then show in Proposition 1 that the equilibrium policy is extreme for any policy function that picks policy "close" to a weak Condorcet winner, and satisfies these three conditions.

[10] In principle, it would be possible to refine away equilibria with more moderate positions for the losing party's candidates, by assuming (for example) that primary voters select the candidate with the closest ideological position to their own ideal one, as long as this does not affect the expected national policy. We discuss uniqueness in more detail in the next subsection.

Let $z(K)$ denote the majority party (i.e., $z(K) = D$ if $\{i|k_i = D\} > \{i|k_i = R\}$, and $z(K) = R$ otherwise), and let $H(K)$ denote the set of majority party legislators, i.e.

$$H(K) = \begin{cases} \{i|k_i = D\} \text{ if } z(K) = D \\ \{i|k_i = R\} \text{ if } z(K) = R \end{cases}$$

The function ξ maps the legislators' ideal positions into an implemented policy. Since we will assume that only the positions of the majority party legislators matter for policy determination, we will write $\xi(x, H)$ in a slight abuse of notation.

We now impose several assumptions on this function ξ.

Assumption 1 *1. Let x and x' be two position vectors that coincide on H (i.e., $x_{i,z(K)} = x'_{i,z(K)}$ for all $i \in H$). Then $\xi(x, H) = \xi(x', H)$.*

2. Let x and x' be two position vectors that permutate ideal positions on H (i.e., there exists a bijective mapping $\beta : H \to H$ such that $x_{i,z(K)} = x'_{\beta(i),z(K)}$ for all $i \in H$). Then $\xi(x, H) = \xi(x', H)$.

The first point in Assumption 1 specifies that only preferences of majority party legislators matter for policy. Essentially, this assumption captures the notion that law-making initiatives are coordinated within the majority party. For example, under the doctrine of the *Hastert Rule* (also called the "majority of the majority rule"), the Speaker of the House will not allow a vote on a bill unless a majority of the majority party supports the bill, even if the majority of the members of the House would vote to pass it.[11] On average, such a rule strengthens the power of majority party legislators, as the minority party cannot pass laws with the votes of a small minority among majority party members, and since the Speaker needs the support of his party to keep his job, this institution appears very stable. See Eguia (2011a,b) for a formal model in which parties arise endogenously to facilitate exactly this type of coordination.

The second point is a symmetry assumption and states that two majority party caucuses that look identical in terms of the positions of legislators implement the same policy. While, in reality, the identity of legislators may matter for how powerful they are within the majority party, this assumption is a useful simplification and appears reasonable in a first model.

[11] See `http://en.wikipedia.org/wiki/Hastert_Rule`.

Assumption 2 states that ξ is Pareto efficient among majority-party legislators, i.e., there is no policy that all majority-party legislators would prefer to $\xi(x, H)$. Given that preferences in our model are spatial, this is equivalent to assuming that ξ selects a policy that is between the most liberal and the most conservative position among majority party legislators.

Assumption 2 $\xi(x, H) \in [\min_{i \in H} x_i, \max_{i \in H} x_i]$.

Finally, Assumption 3 states how the positions of majority party legislators affect policy: Policy is monotone in ideal positions of majority party legislators, both for a fixed set of majority party legislators (e.g., moving the ideal point of a majority party legislator to the right would result in a – possibly very slightly – more right-wing implemented policy) and for a variable set of majority party legislators – say, dropping the most extreme liberal member of the majority party would move the implemented policy to the right.

Assumption 3

1. *$\xi(x, H)$ is continuous in x. Furthermore, $\xi(x, H)$ is strictly monotone in majority party legislators' positions:*

   Suppose that $x' \geq_H x$, i.e. $x'_{i,z(K)} \geq x_{i,z(K)}$ for all $i \in H(K)$, where the inequality is strict for at least one i. Then $\xi(x', H) > \xi(x, H)$.

2. *Let H and $H' = H \setminus \{j\}$ be two sets of majority party legislators (i.e., H' is constructed from H by dropping the legislator from district j). If $x_j \leq x_i$ for all $i \in H'$, then $\xi(x, H') \geq \xi(x, H)$, where the inequality is strict unless $[\min_{i \in H} x_i, \max_{i \in H} x_i]$ is a singleton. Likewise, if $x_j \geq x_i$ for all $i \in H'$, then $\xi(x, H') \leq \xi(x, H)$, where the inequality is strict unless $[\min_{i \in H} x_i, \max_{i \in H} x_i]$ is a singleton.*

It should be clear that there is a non-empty class of functions ξ that are compatible with Assumptions 1, 2 and 3. A rule that always selects the median ideal position of the majority party legislators satisfies Assumptions 1 and 2 and a weak form of the monotonicity Assumption 3. Thus, the function in (9), a small modification of the median rule that satisfies monotonicity, satisfies all three assumptions.

$$\xi(x, H) = \text{median}(\{x_{i,z(K)}\}_{i \in H(K)}) + \delta \left[\Phi \left(\frac{\sum x_i}{\#H(K)} - \text{median} \right) \right], \tag{9}$$

where Φ is any strictly increasing function with $\lim_{t \to -\infty} \Phi(t) = -1$, $\Phi(0) = 0$ and $\lim_{t \to \infty} \Phi(t) = 1$ that satisfies $\Phi' < (n+1)/\delta$.[12]

[12]The bound on Φ' is a sufficient condition for ξ satisfying Assumption 2.

Finally, some of our results focus on the case that the outcome of the law-making process in the majority party is not just Pareto efficient, but "relatively close" to the Condorcet winning policy for that group. If we think of policy being determined by some form of bargaining among the members of the majority party, then the intuition from legislative bargaining models suggests that, for discount factors sufficiently close to 1, the implemented policy should be close to a Condorcet winning policy.

Let $CW(x, H) \subset \mathbb{R}$ denote the set of weak Condorcet winning policies among majority party legislators. Formally, $c \in CW(x, H)$ if and only if, for any c', $\#\{i | i \in H \text{ and } c \succ_{i,z(K)} c'\} \geq \#\{i | i \in H \text{ and } c' \succ_{i,z(K)} c\}$.[13] We now define what we mean by "close."

Definition 2 *A function $\xi: X \times H \to \mathbb{R}$ is δ-close to the set of Condorcet winners (in short,* **δ-close***) if the distance between the implemented policy and the set of weak Condorcet winning policies is no larger than δ, i.e., if*

$$\sup_{x,H} \|\xi(x, H) - CW(x, H)\| \leq \delta.$$

Note that the function in (9) is δ-close because the median is always in the set of weak Condorcet winners (independent of whether $\#H$ is odd or even), and the maximum absolute value of the second term in (9) is δ.

We now state our results for ξ-functions that satisfy Assumptions 1, 1 and 3.

Proposition 1 *Suppose there are $2n+1$ districts, ordered according to the ideology of their general election median voters (i.e., $M_i < M_{i+1}$ for all i), that $n \geq 6$, and that $M_{n+1} \neq 0$. Furthermore, assume that Democratic and Republican primary voters in all districts are located at $-m$ and m, respectively. There exists a $\bar{\delta} > 0$ such that the following holds for any policy selection function that satisfies Assumptions 1–3 and is δ-close to the set of weak Condorcet winners, where $\delta < \bar{\delta}$:*

1. *There exists a pure strategy equilibrium.*
2. *In all equilibria in which the majority party wins a bare majority of exactly $n+1$ seats, the implemented policy is located within δ of the ideal point of the median primary voter of the party preferred by median voter in the median district $n + 1$.*

[13] Clearly, if the number of legislators of the majority party $z(K)$ is odd, then the set of weak Condorcet winning policies is a singleton equal to party $z(K)$'s median legislator's position (i.e., $CW(x, H) = \text{median}(\{x_{i,z(K)}\}_{i \in H})$). In contrast, if the number of majority party legislators is even, then the set of weak Condorcet winning policies is the interval between the two most central positions of party $z(K)$'s legislators.

3. *In all equilibria in which the majority party wins a supermajority of more than* $n + 1$ *seats, the equilibrium implemented policy is located in* $[-m, M_1]$ *if the Democrats win, and* $[M_{2n+1}, m]$ *if the Republicans win.*

6.3 Discussion

Extremism in the legislature. The standard intuition in the existing literature is based on a naive application of the single district model and suggests that two-party competition leads both parties to propose positions close to the ideal of the respective district median voters. Proposition 1 shows that this standard intuition does not carry over to our model, but rather that candidate positions are very extreme in equilibrium. Rather than an assembly of district median voters from all over the country, Proposition 1 predicts a legislature in which the majority of the majority party is at least as extreme as the median voter in the most extreme district that favors the party, and possibly even as extreme as the median primary voter in the majority party. General elections lose their disciplining and moderating power because voters correctly anticipate that both parties are extreme.

Since one of these two extremist parties necessarily wins and therefore controls the legislature, we should not expect that this legislature necessarily adopts policies with broad popular support, as long as they are unpopular with their own party base. For example, an October 7, 2013 Washington Post opinion poll shows that registered voters disapproved of the Republican party shutting down the government by 71 to 26. Thus, it is very likely that the median voters in most districts – even most of those held by Republican House members – opposed shutting down the government, but among voters who identify as Republicans, there was a 52-45 majority in favor of the shutdown, and it is likely that, among those voters who actually vote in Republican primaries, there was an even larger majority in favor of the shutdown.

There are media reports that many Republican representatives "would have liked" to end the government shutdown much sooner, but were afraid that taking this position publicly would put them at risk in their district primary. For example, former House Speaker Dennis Hastert said in an October 7, 2013 interview with NPR: "It used to be they're looking over their shoulders to see who their general [election] opponent is. Now they're looking over their [shoulders] to see who their primary opponent is."

Our model shows that this fear is justified: Primary voters who refuse to renominate a "moderate" and replace him with an extremist are not irrational – because even in a relatively moderate district, an extremist is very likely to win. This makes the primary threat so credible.[14]

Safe and lopsided districts. As mentioned in the introduction, the standard spatial model in which voters look at their district's candidate positions in isolation cannot explain why there are "safe" districts that are essentially guaranteed to be won by one party's candidate. In the equilibria of Proposition 1, the median voters in most (or all) districts have a strict preference for the election winner in their district, implying that winning margins are bounded away from zero, even though there is no uncertainty. Thus, without without appealing to a large "party valence" or "partisanship" that is also unrelated to policy, our model generates districts that are safe and lopsided (i.e., the winner's percentage is significantly larger than 50 percent).

Hence, our model can explain why, for example, a Democrat has a hard time to be elected in Wyoming, even if he adopted a platform that would actually be preferred by the median voter in his district to the platform of the Republican. The contamination effect produces exactly such a result because voters in a biased district are reluctant to vote for the representative of the party whose representatives from other districts are unpopular. For example, a vote for the Democrat in Wyoming would also strengthen the probability that Democrats from other parts of the country get the chance to enact legislation, and their positions are too liberal to be palatable for Wyoming voters.

Bare majority and super-majority equilibria. Proposition 1 distinguishes between two classes of equilibria, "bare majority" (point 2) and "supermajority" equilibria (point 3).

In a "bare majority" equilibrium, the median voter in each district that votes for the majority party, is pivotal for the majority party. Therefore, each of them must prefer the extreme policy position of the majority to what they would get from the minority party. Since the policy position that the minority would implement is no worse than the ideal policy position of its primary voters, this implies that the identity of the party that wins a majority of seats in the legislature depends on the ideological inclinations of the median voter in the median district $n + 1$. Remember that party median positions are normalized in a way that they are at $-m$ and m, i.e., a voter at 0 is indifferent between these positions. If the median voter in the median

[14] Alternative explanations for non-median policy outcomes include lobbying and differential preference intensity. Arguably, neither of these alternative explanations is plausible for the government shutdown.

The lobbying explanation (a strong lobby in favor of the minority position is able to "buy" the support of legislators)requires that benefits on the minority side are highly concentrated, while the issue is a relatively minor issue for most voters. The intensity explanation requires that minority voters care more about the issue, but according to the same Washington Post poll cited above, 12 percent of registered voters "strongly approve" of the shutdown, 14 percent "approved somewhat," while 53 percent "strongly disapprove" and 18 percent "disapprove somewhat." Thus, intensity about the issue appears higher among those who disapprove.

district prefers the Democratic party position to the Republican one, then, in a competitive equilibrium, Democrats win a majority of seats, and vice versa.

In a supermajority-equilibrium, no district is pivotal for which party gets a majority in the legislature. The only effect of a district's choice is on the majority party's composition. For this reason, there is a continuum of equilibrium policies in this class of equilibria. In these equilibria, all majority party legislators are located at the same position, and this position can be anywhere between the ideal point of the most extreme district's median voter, and the ideal point of the party primary median voters.

A deviation of a general election median voter to the other party's candidate then does not affect the implemented policy, a deviation of the majority party primary median voter to a more extreme position is not successful (because the general election voter in that district is then better off with the opposition candidate), and a deviation of the majority party primary median voter to a more moderate position is not desirable from his perspective because it would lead to a more moderate implemented policy.

Interestingly, the equilibrium policy is potentially more moderate in a supermajority equilibrium, which contradicts the simplistic intuition that a more competitive situation should necessarily result in a more moderate equilibrium outcome. The closeness of the seat count in the bare majority equilibrium is exactly what generates more extremism there than in most supermajority equilibria. Since, in a bare majority equilibrium, each of the median voters that vote for the majority party in equilibrium knows that, if he switches his vote, his ideologically closer party is replaced by the other party, he is very reluctant to do that: The "threat" to vote for the opposition candidate if the candidate of the favored party is too extreme is not credible. In contrast, in a supermajority equilibrium, switching to the opposition candidate in any one district has very limited policy implications as the majority party remains the same, and therefore provides for a much more credible threat.

7 The Effect of Contamination on Competitive Districts

In the model without uncertainty analyzed in the last section, all districts are safe for one of the parties, and parties nominate extreme candidates. We now address what happens with competitive districts where both candidates have a strictly positive probability of winning. In this case, parties have a non-trivial trade-off between nominating ideologically close candidates and nominating moderates who have a better change of winning the election, and thus giving the party a majority in the legislature.

This modification of the model is important in order to take the model closer to the data: In reality, there are some "swing districts" in which both Democrats and Republicans are competitive, and the behavior of representatives from these swing districts plays a key role in the empirical analysis of the effects of "gerrymandering." How does the creation of districts that are internally ideologically homogeneous (either very conservative or very liberal) and in which representatives are conceivably much more afraid of a primary challenge from within their own party than of the opposition candidate in the general election affect equilibrium policy divergence between parties?

Many observers suspect that increased polarization in Congress is *caused* by gerrymandering. However, there is an empirical literature in political science that claims that gerrymandering has no significant effect on polarization between the two parties in Congress, arguing that polarization and gerrymandering increase at the same time, but that there is no causal relationship between the two. In this view, the principal driver of polarization between parties is "sorting" in the sense that conservative districts are increasingly represented by Republicans. McCarty et al. (2009) use the DW-Nominate score of representatives as their dependent variable, and analyze how it correlates with district characteristics (in particular, the district's PVI as a proxy of its ideological bent) and the party affiliation of the district representative. They argue (p. 667) that "for a given set of constituency characteristics, a Republican representative compiles an increasingly more conservative record than a Democrat does. Gerrymandering cannot account for this form of polarization" because this change in behavior occurs even in swing districts.

Implicitly, this argument assumes that the only effect of gerrymandering is on the equilibrium positions of candidates in those districts that are directly affected: For example, if a district is gerrymandered to be more conservative, then positions of candidates *in that district* will be more conservative, but there are no spill-over effects on the positions of candidates in other districts that remain moderate. In order to analyze whether this is a logically justified argument, we extend the framework of the last section to allow for uncertainty about the district median voter positions.

Consider again $2n + 1$ districts, with primary voters located at $-m$ and m, respectively. In districts 1 to k_D the median voters are located to the left of zero, and to the right of zero in districts $2n + 1 - k_R$ to $2n + 1$, with probability 1 — in equilibrium these will be the safe districts. In the remaining "competitive" districts, the median voter is i.i.d. with a cdf $\Phi(x)$. We focus on equilibria that are symmetric across competitive districts, i.e., $x_{D,i} = x_D$ and $x_{R,i} = x_R$ for all $k_D < i < 2n + 1 - k_R$.

Furthermore, we suppose that the implemented policy is determined by the median position of the majority party legislators. Voters also care, with a very small weight γ, about their local representative's position (so their utility function is of the form of (1) and (2), with $\gamma \to 0$). This assumption simplifies some proofs, but the results below also hold for more general mappings ξ from policy positions to a party's policy as long as the influence of an individual representative on the policy is sufficiently small, and as long as ξ is sufficiently δ-close to the set of weak Condorcet winners in the sense of Definition 2.

Proposition 2 analyzes the case that $k_D, k_R < (n-1)/2$ so that, since the majority party has at least $n+1$ representatives, a supermajority of the majority party's representatives come from competitive districts. Thus, even if a legislator from a competitive districts deviates, the majority of legislators is located at x_D or x_R, respectively.

Proposition 2 *Suppose that $k_D, k_R < (n-1)/2$. Let Φ_0 be a symmetric distribution with mean zero. Suppose that the distribution of the median voter in each competitive district is i.i.d. with distribution $\Phi(x) = \Phi_0(x - M)$, and that $\xi(x, H) = \text{median}(\{x_{i,z(K)}\}_{i \in H(K)})$ (i.e., the implemented policy is equal to the median majority legislator's position).*

Then, for any M in a neighborhood of zero and for small γ, there exists a unique equilibrium that is symmetric across competitive districts, i.e., $x_{D,i} = x_D(M)$, and $x_{R,i} = x_R(M)$ for all districts i with $k_D < i < 2n + 1 - k_R$.

1. *$x_D(M)$ and $x_R(M)$ are independent of the number of competitive districts and independent of γ. Furthermore, $x_D(M) \leq x_R(M)$ for all M.*
2. *$x_D(0) = -\frac{m}{1+2\phi_0(0)m}$, and $x_R(0) = \frac{m}{1+2\phi_0(0)m}$.*
3. *$x_R(M) - x_D(M) < x_R(0) - x_D(0)$ for $M \neq 0$ in a neighborhood of zero.*
4. *The probability that R wins is strictly increasing in M for M in a neighborhood of zero.*
5. *Districts $i \leq k_D$ are safe for Democrats, who get elected with position $-m$, while districts $2n + 1 - k_R$ to $2n + 1$ are safe for Republicans who get elected with position m.*

Proposition 2 has interesting implications for the effects of gerrymandering. Remember that we normalize such that the voter located at 0 is indifferent between the positions of the Democratic and Republican primary voters at $-m$ and m, respectively. Start from a situation where all districts are identical and the expected median voter M is located close to 0, and suppose now that district boundaries are redesigned such that there are some Democratic leaning district in which the median voters is always to the left of zero, and some Republican districts where the median voter is strictly to the right of zero. According to Proposition 2 these districts are safe for the Democrats and Republicans, respectively. In the remaining competitive districts, there could be potentially three distinct effects.

First, the gerrymander may shift the distribution of median voters in competitive districts. For example, if both parties have the same number of safe districts, but the Republican safe districts are more moderate than the Democratic ones, then the expected median voter M in the competitive districts shifts to the right, and Proposition 2 indicates how this impacts the elections there. The winning probability for the Democrat decreases, and if originally $M < 0$, then candidates are more polarized, while the reverse is true if $M > 0$.

Second, if one party has more of the safely-gerrymandered districts than the other party, then it has an obvious advantage in winning a majority of the legislature, since it needs to win fewer of the competitive districts. Potentially, this increased winning probability could affect the behavior in the competitive districts, but interestingly, Proposition 2 shows that it does not, and the same is also true in the case discussed below that representatives from the gerrymandered districts affect policy.

Third, more extreme legislators of the same party can "contaminate" the candidates in the swing districts, as discussed above. However, this effect is not present in Proposition 2 because, by assumption, there are only few gerrymandered districts, so that the members from those districts have no influence on national policy. In Proposition 3, we modify this assumption to analyze the case where the policy is determined by the candidates elected from safe districts.

In Proposition 3, we assume that the median voters in the safe districts are at $-m$ and m respectively so that there is a unique equilibrium position for the winners in these districts at $-m$ and m, respectively. We do this to simplify the proof, and because the point is anyway only to provide a possibility result.[15]

Proposition 3 *Make the same assumptions as in Proposition 2, except that now $k_D, k_R > (2n + 1)/3$, and*

[15] Remember that Proposition 1 suggests the possibility of a continuum of positions in supermajority equilibria. It is unclear whether such a possibility would in fact arise in the setting used here, because the key to the more moderate equilibria is the general election median voter's threat to vote for the other party if his preferred party nominates a candidate who is too extreme. If there is uncertainty about the election outcome, then voting for one's less preferred party is much more costly, because such an outcome in a non-swing district may actually switch the majority in the legislature. Therefore, we conjecture that the winning candidates' equilibrium positions in safe districts would be close to $-m$ and m anyway, even if the median voters' ideal positions are in these districts are more moderate. Our assumption allows us to avoid a formal proof of this conjecture, which is beneficial because our main focus is on the equilibrium behavior in competitive districts.

that the district median voters are at $-m$ for $i \leq k_D$ and m for $i \geq 2n+1-k_R$.

Then for any M in a neighborhood of zero and for small γ, there exists a unique equilibrium that is symmetric across competitive districts, i.e., $x_{D,i} = x_D(M)$, and $x_{R,i} = x_R(M)$ for all districts i with $k_D < i < 2n+1-k_R$.

1. *The candidates nominated in the competitive districts are given by*

$$x_D = -\frac{\Phi_0(-M)m}{\Phi_0(-M) + \phi_0(-M)m}, \quad x_R = \frac{(1-\Phi_0(-M))m}{(1-\Phi_0(-M)) + \phi_0(-M)m}. \tag{10}$$

2. *There exists $\bar{m} < \infty$ such that for all $M \neq 0$, and $m > \bar{m}$ polarization is larger than in the case with few gerrymandered district from Proposition 2.*

Proposition 3 shows that polarization in the other districts affects equilibrium positions in the competitive districts. Provided that the difference between party ideal positions is sufficiently large, candidate position are further apart than in the case with few gerrymandered districts (the only exception to this is the non-generic case in which the expected median M is exactly zero).

This is a surprising result. A superficial intuition would suggest that, if party ideal points are far apart from each other, each party should be very concerned with the possibility of the other party taking over the majority in the legislature, and therefore should do its utmost in order to compete in the moderate swing districts, by nominating very moderate candidates there.

This is certainly true, but only one part of the intuition. The countervailing force is that, if party positions are far apart from each other, then the position of the cutoff voter in the general election (i.e., the one who is indifferent between the two local candidates) becomes very inelastic with respect to their positions, as he understands that the main potential effect of his local choice is the chance that it affects the identity of the majority party. The less elastic the cutoff voter reacts to changes in local candidates' positions, the more the parties have an incentive to nominate candidates who are close to their respective ideal positions. For $M = 0$, these two effects exactly cancel while for $M \neq 0$, the inelasticity effect actually outweighs the effect that winning the election is more important for parties.

Our results show that gerrymandering a particular district does not just affect that district, but other districts as well. In particular, if the more extreme legislators from gerrymandered districts determine the national policy, then we should observe increased polarization in the remaining competitive districts, exactly the behavior noted by the empirical literature. Of course, the same arguments also apply to the Senate where there is obviously no “gerrymandering,” but where increased regional preference differences have created an increasing number of safe seats for the parties. More extreme candidates elected in these safe states impose an externality on the remaining competitive states, creating increased polarization in those states as well.

8 Discussion

Much of the existing literature on electoral competition in legislative elections implicitly assumes that voters evaluate their local candidates based on their positions, but not on the party label under which they run. Such a model implies that both parties nominate candidates who are very close to the preferences of the respective district median voters. Therefore, even in districts with rather extreme preferences, both parties’ candidates should be competitive, and the position of Democratic and Republican Congressmen elected from similar districts should be very similar. It is safe to say that these predictions are not borne out in reality, and to understand why this is the case is of first-order importance for our understanding of the American democratic system.

In this paper, we have developed a theory of candidate nomination processes predicated upon the notion that majority party legislators collaboratively influence policy. This assumption is appears reasonable and yields fundamentally different results.

In our model, a candidate’s association with candidates of the same party that run in other districts generates an incentive for voters to focus less on the candidates’ own position positions when deciding whom to vote for — local candidates are “contaminated” by their party association. This leads to less competitive local elections, providing the ideologically favored party with the leeway to nominate more extreme candidates who are nevertheless elected. As a consequence, the equilibrium of our model can explain how electoral competition can beget a very polarized legislature.

Our analysis has two additional important empirical implications. First, it can explain why a district’s ideological preferences have a smaller partisan effect in elections in which a candidate has a more autonomous policy influence, such as elections for executive leadership positions than in legislative elections. Of course, in reality, even executive leader positions are not entirely autonomous, so there will be some contamination in executive elections as well, but we would expect this effect to be smaller than in legislative elections, and this expectation is borne out in our empirical analysis of Senate and Gubernatorial elections in Section 3.

Second, much of the existing empirical analysis of the effects of gerrymandering on polarization in Congress is implicitly based on applying a naive model in which voters care only about the local candidates' positions. Such a model may lead to incorrect inferences about the importance of gerrymandering. For example, the ideal position of the district median voter often does not affect the equilibrium position of candidates at the margin in our model, but the total effect of gerrymandering on polarization in Congress may nevertheless be substantial (and actually be much larger than in the naive model). Thus, one cannot infer that gerrymandering does not matter for polarization in Congress from showing that there is no marginal effect of changes in district medians on ideological positions of legislators, and that the difference in voting records of Republicans and Democrats representing the same or very similar districts has increased. In general, an implication of our model for empirical work is that legislator behavior in different districts is intricately connected rather than independent, and this implies that one needs to be very careful with claims that difference-in-difference approaches can identify causation.

9 Appendix

9.1 Proofs

Proof of Proposition 1.

1. We prove existence by construction. Assume first that $M_{n+1} < 0$, and consider the following profile: Democrats nominate candidates located at $-m$ in every district, and Republicans nominate candidates located at m in every district. As a consequence, all voters with $\theta < 0$ vote for the Democratic candidate, and all voters with $\theta > 0$ vote for the Republican candidate, and Democrats win a majority in the legislature.

   Clearly, voters behave optimally given the candidate positions, and Democratic primary voters do not have an incentive to deviate because they receive their ideal policy. Furthermore, if Democrats win a majority larger than just one seat, then a deviation by Republicans in any district does not change the national policy. Thus, we only have to consider the case in which Democrats win by exactly one seat, and we can focus on a deviation by Republicans in district $n+1$. Given such a deviation, if the median voter in district $n+1$ elects the Republican, the national policy would be $\xi \geq m - \delta$. However, as $M_{n+1} < 0$, for all $\delta < 2|M_{n+1}|$, the median voter M_{n+1} continues to prefer the policy $-m$ that results when electing the Democrat. Thus, no successful deviation is feasible for Republicans in district $n+1$. The same is evidently true in all districts $i < n+1$. Thus, the profile described is an equilibrium. If $M_{n+1} > 0$, the construction of the equilibrium is analogous.

2. Consider the case that the winning party wins a majority of exactly one seat. We want to prove that there is no equilibrium in which the resulting policy is more moderate than within δ of the primary median voter positions $-m$ or m.

   Each median voter in a district that elects a member of the majority party (without loss of generality, assume again that this is the Democrats) is pivotal and therefore must, at least weakly, prefer the policy $\tilde{\xi}$ to the policy that would result from electing the Republican local candidate.

   In all districts i where this preference is strict, we must have $x_{i,D} = -m$, because, if $x_{i,D} > -m$ then the median Democratic primary voter in district i prefers a lower $x_{i,D}$, by monotonicity of ξ; moreover, by continuity of ξ, such a candidate with a slightly lower $x_{i,D}$ would still be strictly preferred by the general election median voter, and thus be elected.

It remains to prove that we cannot have a majority of those districts j in which Democrats win, have a median voter who is indifferent between the Democratic equilibrium policy $\tilde{\xi}$ and the Republican policy ξ_j^R that would result if district j were to elect its Republican candidate.

For any Republican candidate from a districts in which the Democrats win in equilibrium, there exists an interval of "implementable" policies $\Xi = (\underline{\xi}^R, \overline{\xi}^R)$; that is, for each policy $\hat{\xi} \in \Xi$, there exists a position for the local Republican candidate such that, if he was elected with this position and added to the Republican caucus so that they form a majority, the resulting policy would be $\hat{\xi}$.

Consider a district in which the median voter is indifferent between the Democrat and the Republican. The Republican candidate from this district must be located at a position that implements the policy from Ξ that is optimal for the district median voter (if this was not the case, then the Republicans could choose a slightly better candidate and win in the district, and consequently, win a majority in the legislature).

We now argue that there can be at most three "indifferent districts" (i.e., those, in which the median voter is indifferent between the Democrat and the Republican).

Consider the following partition of the real line: $(-\infty, \underline{\xi}^R]$, $(\underline{\xi}^R, \overline{\xi}^R)$, $[\overline{\xi}^R, \infty)$. If there are more than three indifferent districts, then at least one of these sets contains at least two different median voters of indifferent districts.

If they are in $(-\infty, \underline{\xi}^R]$, then the corresponding Republican candidates must both choose an effective policy as close as possible to $\underline{\xi}^R$, and both of the median voters must be indifferent between $\underline{\xi}^R$ and the Democratic equilibrium policy $\tilde{\xi}$. This is impossible since the median voters have different ideal points smaller than $\underline{\xi}^R$. An analogous argument excludes the case that there are median voters from two or more indifferent districts in $[\overline{\xi}^R, \infty)$. Finally, if there are two median voters from indifferent districts in $(\underline{\xi}^R, \overline{\xi}^R)$, then the Republican candidates can each offer these median voters their respective ideal policies, and thus, it is impossible that both of them are indifferent between their respective ideal policies and the Democratic equilibrium policy (which must be the same in both districts). In summary, these contradictions prove that there cannot be more than 3 indifferent districts. Since a majority caucus consists of at least 7 members, the result follows.

3. We now consider equilibria in which one party wins more than $n+1$ seats; without loss of generality,

suppose that the winners are the Democrats.

It is straightforward to see that any profile in which all Democrats are located at any $\tilde{\xi} < M_1$ is an equilibrium: First, electing a Republican in any one district in which Democrats win in equilibrium has no effect on implemented policy and thus does not change voters' utility. Second, since the preceding argument is independent of the positions of Republican candidates, we do not need to worry about deviations by Republican primary voters. Last, consider a deviation by a Democratic primary voter. If he nominates a more extreme candidate who would move policy to the left (because of monotonicity of ξ), then the median voter in the general election would be better off electing the Republican candidate, since that leaves the policy at $\tilde{\xi}$. If, instead, he nominates a more moderate candidate, the median voter in the general election would be happy to vote for that candidate, but it would move national policy toward a more moderate position, contrary to the interests of the Democratic primary voter.

Assume now that, to the contrary of the claim, there exists an equilibrium in which $\xi(x, H) = \tilde{\xi} > M_1 + \delta$.

Suppose first that all Democrats who win in this equilibrium are located at the same position $\tilde{\xi} > M_1 + \delta$. Consider a deviation by the Democratic primary voter in district 1 to a slightly more left-leaning candidate located at $\tilde{\xi} - \varepsilon$. By monotonicity of ξ, if this candidate is elected, the policy moves to the left, which is preferred by both the median voter in district 1 and by the Democratic primary voter. Thus, the original profile was not an equilibrium.

Consider now the other case, namely that not all Democrats who win in this equilibrium are located at the same position, and let i be a district with a Democrat that is located at the most moderate position among Democrats (i.e., $x_{i,D} \geq x_{j,D}$ for all $j \in H$, where the inequality is strict for some j).

By Assumption 3, if district i elects the Republican, the remaining Democrats (who are still in the majority) will implement a policy $\xi' < \tilde{\xi}$. There are two possible cases: If $M_i \geq \tilde{\xi}$, then the median voter in district i would be strictly worse off if he elected the Republican; moreover, continuity of ξ in $x_{i,D}$ implies that there are some slightly more leftist platforms for the Democratic candidate in district i such that the median voter of district i still prefers the policy when electing the Democrat over the policy when electing the Republican. Since the district i Democratic primary voter's utility increases when $x_{i,D}$ moves to the left and the Democrat is still elected, the original profile was not an equilibrium.

If $M_i < \tilde{\xi}$, then, by continuity and monotonicity of ξ, a small decrease in $x_{i,D}$ is beneficial for both the median voter in district i and for the Democratic primary voter in district i. Thus, the original strategy profile was not an equilibrium.

■

Proof of Proposition 2. Consider a particular competitive district, and let p_k be the probability that k of the remaining $2n$ districts vote Republican. Suppose that the Republican in district i deviates to policy y. Since $k_R < (n-1)/2$ the median policy of if the Republicans win remains x_R. The the payoff of a voter at M from the Democrat is

$$-(1-\gamma)\left(\sum_{k=0}^{n} p_k(M-x_D)^2 + \sum_{k=n+1}^{2n} p_k(M-x_R)^2\right) - \gamma(M-x_D)^2,$$

while the payoff from the Republican is

$$-(1-\gamma)\left(\sum_{k=0}^{n-1} p_k(M-x_D)^2 + \sum_{k=n}^{2n} p_k(M-x_R)^2\right) - \gamma(M-y)^2.$$

Thus, we can conclude that the cutoff voter is given by

$$M_R(x_D, x_R, y) = \frac{1}{2}\frac{(1-\gamma)p_n(x_R^2 - x_D^2) + \gamma(y^2 - x_D^2)}{(1-\gamma)p_n(x_R - x_D) + \gamma(y - x_D)}. \tag{11}$$

It follows immediately that

$$M_R(x_D, x_R, x_R) = \frac{x_D + x_R}{2}, \quad \left.\frac{\partial M_R(x_D, x_R, y)}{\partial y}\right|_{y=x_R} = \frac{1}{2}\frac{\gamma}{(1-\gamma)p_n + \gamma}. \tag{12}$$

Suppose by contradiction that $x_R(\bar{M}) < x_D(\bar{M})$. In this case liberals vote for the Republican and conservatives for the Democrat. The cutoff voter is located at $(x_D(M) + x_R(M))/2$. Now suppose the Republican's position is changed to $x_{R,i} = -M$. Then (12) implies that the cutoff voter becomes more liberal. Hence the probability that the Democrat is elected in district i increases. Thus, the probability that Democrats receive a majority in the legislature and policy $x_D(\bar{M})$ is implemented increase, making the Republican primary voter at m strictly better off, as long as γ is sufficiently small, a contradiction. Hence $x_R(\bar{M}) \geq x_D(\bar{M})$.

Since $\Phi(M_R(x_D, x_R))$ is the probability that the Democrat gets elected, the Republican primary voter solves

$$\max_y -\Phi\left(M_R(x_D, x_R, y)\right)\left((1-\gamma)\left(\sum_{k=0}^{n} p_k(m-x_D)^2 + \sum_{k=n+1}^{2n} p_k(m-x_R)^2\right) + \gamma(m-x_D)^2\right)$$
$$- \left(1-\Phi\left(M_R(x_D, x_R, y)\right)\right)\left((1-\gamma)\left(\sum_{k=0}^{n-1} p_k(m-x_D)^2 + \sum_{k=n}^{2n} p_k(m-x_R)^2\right) + \gamma(m-y)^2\right). \tag{13}$$

The first derivative with respect to y is given by

$$-\phi\left(M_R\right)\frac{\partial M_R}{\partial y}\left((1-\gamma)p_n\left((m-x_D)^2-(m-x_R)^2\right)+\gamma\left((m-x_D)^2-(m-y))^2\right)\right)$$
$$+\left(1-\Phi\left(M_R\right)\right)2\gamma(m-y) \tag{14}$$

The second derivative is

$$-\left(\phi\left(M_R\right)\frac{\partial^2 M_R}{\partial y^2}+\phi'\left(M_R\right)\left(\frac{\partial M_R}{\partial y}\right)^2\right)\left((1-\gamma)p_n\left((m-x_D)^2-(m-x_R)^2\right)+\gamma\left((m-x_D)^2-(m-y))^2\right)\right)$$
$$-4\gamma\phi\left(M_R\right)\frac{\partial M_R}{\partial y}(m-y)-2\gamma\left(1-\Phi\left(M_R\right)\right). \tag{15}$$

Equation (11) implies

$$\lim_{\gamma\to 0}\frac{1}{\gamma}\frac{\partial M_R(x_D, x_R, y)}{\partial y} = -\frac{x_D+x_R-2y}{p_n(x_R-x_D)}, \quad \lim_{n\to\infty}\frac{1}{\gamma}\left(\frac{\partial M_R(x_D, x_R, y)}{\partial y}\right)^2 = 0, \tag{16}$$

and

$$\lim_{n\to\infty}\frac{1}{\gamma}\frac{\partial^2 M_R(x_D, x_R, y)}{\partial y^2} = \frac{1}{p_n(x_R-x_D)}. \tag{17}$$

Dividing (15) by γ, taking the limit for $n \to \infty$ and using (16) and (17)yields

$$-\frac{\phi\left(M_R\right)}{p_n(x_R-x_D)}p_n\left((m-x_D)^2-(m-x_R)^2\right)-\left(1-\Phi\left(M_R\right)\right)2\gamma < 0, \tag{18}$$

since $x_D < x_R$. Thus, the second order condition is satisfied for all y when γ is small.

Evaluating the first order condition (14) at $y = x_R$ yields (after canceling the common terms and multiplying by 2)

$$-\phi\left(\frac{x_D + x_R}{2}\right)\left((m - x_D)^2 - (m - x_R)^2)\right) + \left(1 - \Phi\left(\frac{x_D + x_R}{2}\right)\right)4(m - x_R) = 0. \tag{19}$$

Similarly, the first order condition for a Democratic primary is (again after canceling and multiplying)

$$-\phi\left(\frac{x_D + x_R}{2}\right)\left((m + x_D)^2 - (m + x_R)^2)\right) - \Phi\left(\frac{x_D + x_R}{2}\right)4(m + x_D) = 0. \tag{20}$$

We now show that the first order conditions have a unique solution at $M = 0$.

Rearranging (19) and (20), we get

$$\phi\left(\frac{x_D + x_R}{2}\right) = \frac{\left(1 - \Phi\left(\frac{x_D+x_R}{2}\right)\right)4(m - x_R)}{\left((m - x_D)^2 - (m - x_R)^2)\right)} = \frac{\Phi\left(\frac{x_D+x_R}{2}\right)4(m + x_D)}{-\left((m + x_D)^2 - (m + x_R)^2)\right)} \tag{21}$$

Note that the denominator of the term in the middle of (21) can be written $(x_R - x_D)(2m - x_R - x_D)$, and the denominator of the right term of (21) can be written $(x_R - x_D)(2m + x_R + x_D)$.

Substituting this and canceling common terms, we get

$$\left(1 - \Phi\left(\frac{x_D + x_R}{2}\right)\right)(m - x_R)(2m + x_R + x_D) = \Phi\left(\frac{x_D + x_R}{2}\right)(m + x_D)(2m - x_R - x_D) \tag{22}$$

Suppose that $x_R + x_D > 0$. Since, for $M = 0$, Φ is symmetric, this implies that $\Phi\left(\frac{x_D+x_R}{2}\right) > \frac{1}{2} > 1 - \Phi\left(\frac{x_D+x_R}{2}\right)$. For (22) to hold, we must therefore have

$$(m - x_R)(2m + x_R + x_D) > (m + x_D)(2m - x_R - x_D), \tag{23}$$

which simplifies to $x_R^2 < x_D^2$, and hence $|x_R| < |x_D|$. However, as shown above $x_D < x_R$. Thus, $x_D + x_R < 0$, a contradiction. Similarly we get a contradiction if we assume that $x_R + x_D < 0$. Hence $x_D = -x_R$.

The first order conditions together with the fact that $x_D = -x_R$ now imply we can simplify the equation to get

$$2\phi(0)x_R m = 2(1 - \Phi(0))(m - x_R).$$

Since $\Phi(0) = 1/2$ we get

$$x_R = \frac{m}{1 + 2\phi(0)m}, \tag{24}$$

Now recall that $\Phi(x) = \Phi_0(x - M)$ and $\phi(x) = \phi_0(x - M)$. At $M = 0$ strategies are symmetric around zero and hence $x_D + x_R = 0$. We now take the derivatives of (19) and (20) with respect to M, evaluated at $M = 0$. To shorten the notation we write x'_R and x'_D for $x'_R(0)$ and $x'_D(0)$.

$$\begin{aligned} &-\phi'_0(0)\left(\frac{x'_D + x'_R}{2} - 1\right)\left((m - x_D)^2 - (m - x_R)^2)\right) - \phi_0(0)\left(-2(m - x_D)x'_D + 2(m - x_R)x'_R\right) \\ &-\phi_0(0)\left(\frac{x'_D + x'_R}{2} - 1\right)4(m - x_R) - (1 - \Phi_0(0))4x'_R = 0, \end{aligned} \tag{25}$$

and

$$\begin{aligned} &-\phi'_0(0)\left(\frac{x'_D + x'_R}{2} - 1\right)\left((m + x_D)^2 - (m + x_R)^2)\right) - \phi_0(0)\left(2(m + x_D)x'_D - 2(m + x_R)x'_R\right) \\ &-\phi_0(0)\left(\frac{x'_D + x'_R}{2} - 1\right)4(m + x_D) - \Phi_0(0)4x'_D = 0. \end{aligned} \tag{26}$$

If $M = 0$ we have the symmetric equilibrium characterized above where x_R is given by (24). Thus, (25) and (26) imply

$$x'_D(0) = x'_R(0) = \frac{4\phi_0(0)^2 m^2}{4\phi_0(0)^2 m^2 + 1}. \tag{27}$$

The second derivatives of (19) and (20) with respect to M evaluated at $M = 0$ are

$$-4\phi''_0(0)mx_R\left(\frac{x'_D + x'_R}{2} - 1\right)^2 - 4\phi_0(0)\left((m - x_R)x''_R - x_R x''_D - 2\left(\frac{x'_D + x'_R}{2} - 1\right)x'_R\right) - 2x''_R = 0; \tag{28}$$

$$4\phi''_0(0)mx_R\left(\frac{x'_D + x'_R}{2} - 1\right)^2 - 4\phi_0(0)\left((m - x_R)x''_D - x_R x''_R + 2\left(\frac{x'_D + x'_R}{2} - 1\right)x'_D\right) - 2x''_D = 0. \tag{29}$$

(28) and (29) imply $x''_D(0) = -x''_R(0)$. Let $S = 0.5(x'_D(0) + x'_R(0)) - 1$. Then (27) implies $S < 0$. We can solve (28) for x''_R to get

$$x''_R(0) = \frac{2\phi''_0(0)mx_R S^2 + 4\phi_0(0)x'_R(0)S}{1 + 2\phi_0(0)m}. \tag{30}$$

Thus, $x''_R(0) < 0$. Since $x''_D(0) = -x''_R(0)$ it follows that $x''_R(0) - x''_D(0) < 0$. As a consequence $x_R(M) - x_D(M)$ assumes a local maximum at $M = 0$. Therefore $x_R(\bar{M}) - x_D(\bar{M}) < x_R(0) - x_D(0)$ for $\bar{M} \neq 0$ in a neighborhood of 0.

Finally, note that $x_D < 0 < x_R$ near $M = 0$, which implies that the median voters in districts 1 to k_D strictly prefer that the Democrats win, while Republicans in districts $2n+1-k_R$ to $2n+1$ strictly prefer that the Republicans win. As a consequence, districts 1 to k_D are safe for the Democrat, who gets elected with policy $-m$, while districts $2n+1-k_R$ to $2n+1$ are safe for the Republicans who get elected with policy m if γ is not too large.

In particular, suppose that the median voter in district $i < k_D$ deviates and elects the Republican. Then the probability that policy x_R is implemented increase, while the the probability of policy x_D decreases, which makes the median voter worse off as long as γ is small. Since the median voter is strictly better off with the Democrat, the primary voter will therefore propose a candidate with policy $x_{D,i} = -M$. The argument that districts $2n+1-k_R$ to $2n+1$ are safe for the Republican is analogous. ■

Proof of Proposition 3. Since the median voters in the gerrymandered districts are at $-m$ and m, respectively, the resulting policies in these districts are $-m$ and m. By assumption the gerrymandered districts are at least 2/3 of all districts. As a consequence, the median legislature is at $-m$ if the Democrat win, and at m if the Republicans win.

Consider a particular competitive district, and let p_k be the probability that k of the remaining $2n$ districts vote Republican. Denote the Democrat's and the Republican's policies by x_D and x_R. Then the payoff of a voter at M from the Democrat is

$$-(1-\gamma)\left(\sum_{k=0}^{n} p_k(M+m)^2 + \sum_{k=n+1}^{2n} p_k(M-m)^2\right) - \gamma(M-x_D)^2,$$

while the payoff from the Republican is

$$-(1-\gamma)\left(\sum_{k=0}^{n-1} p_k(M+m)^2 + \sum_{k=n}^{2n} p_k(M-m)^2\right) - \gamma(M-y)^2.$$

The cutoff voter, who is indifferent between the candidates, is therefore given by

$$M(x_D, x_R) = \frac{1}{2}\frac{\gamma(x_R^2 - x_D^2)}{\gamma(x_R - x_D) + 2(1-\gamma)p_n m}. \tag{31}$$

Note that

$$\lim_{\gamma\to 0}\frac{1}{\gamma}\frac{\partial M(x_D, x_R)}{\partial x_D} = -\frac{x_D}{2p_n m}, \text{ and } \lim_{\gamma\to 0} M(x_D, x_R) = 0. \tag{32}$$

The Democratic primary voter therefore solves

$$\max_{x_D} -\Phi\left(M_R(x_D, x_R)\right)\left((1-\gamma)\sum_{k=n+1}^{2n} p_k(2m)^2 + \gamma(m+x_D)^2\right)$$
$$-\left(1-\Phi\left(M_R(x_D, x_R)\right)\right)\left((1-\gamma)\sum_{k=n}^{2n} p_k(2m)^2 + \gamma(m+x_R)^2\right).$$

The first order condition is

$$-\phi(M)\frac{\partial M(x_D, x_R)}{\partial x_D}\left(\gamma((m+x_D)^2-(m+x_R)^2)-4(1-\gamma)p_n m^2\right)-2\Phi(M)\gamma(m+x_D))=0. \quad (33)$$

Dividing both sides of (33) by γ, then taking the limit for $\gamma \to 0$, and using (32) yields

$$\phi(0)(-x_D)m = \Phi(0)(m+x_D). \quad (34)$$

The Republican primary solves

$$\max_{x_R} -\Phi\left(M_R(x_D, x_R)\right)\left((1-\gamma)\sum_{k=0}^{n} p_k(2m)^2 + \gamma(m-x_D)^2\right)$$
$$-\left(1-\Phi\left(M_R(x_D, x_R)\right)\right)\left((1-\gamma)\sum_{k=0}^{n-1} p_k(2m)^2 + \gamma(m-x_R)^2\right).$$

The first order condition is

$$-\phi(M)\frac{\partial M(x_D, x_R)}{\partial x_R}\left((1-\gamma)4p_n m^2+\gamma((m-x_D)^2-(m-x_R)^2)\right)+2\gamma\left(1-\Phi(M)\right)(m-x_R)=0.$$

It follows that

$$\frac{\partial}{\partial x_R}\frac{M(x_D, x_R)}{\gamma}\Big|_{\gamma=0} = -\frac{x_R}{2p_n m}.$$

Again, dividing by γ, setting $\gamma = 0$ and using the fact that $M = 0$ when $\gamma = 0$, yields

$$\phi(0)x_R m = \left(1-\Phi(0)\right)(m-x_R). \quad (35)$$

This implies (10).

We now show that that the objectives of the Democrats' maximization problems is strictly concave. The derivative of (33) is

$$-\left(\phi(M)\frac{\partial^2 M(x_D,x_R)}{\partial x_D^2}+\phi'(M)\left(\frac{\partial M(x_D,x_R)}{\partial x_D}\right)^2\right)\left(\gamma\left((m+x_D)^2-(m-x_R)^2\right)-(1-\gamma)4p_n m^2\right) \\ -4\gamma\phi(M)\frac{\partial M(x_D,x_R)}{\partial x_D}(m+x_D)-2\gamma\phi(M). \tag{36}$$

Note that

$$\lim_{\gamma\to 0}\frac{\partial^2 M(x_D,x_R)}{\partial x_D^2}\frac{1}{\gamma}=-\frac{1}{2p_n m},\ \text{ and }\ \lim_{\gamma\to 0}\left(\frac{\partial^2 M(x_D,x_R)}{\partial x_D^2}\right)^2\frac{1}{\gamma}=0. \tag{37}$$

Dividing both sides of (36) by γ, taking the limit for $\gamma \to 0$, and using (32), and (37) yields $-2\phi(M)(m+1) < 0$. Thus, for small γ the objectives is concave for every x_D. Concavity of the Republican's objective follows similarly.

If $M = 0$ then $\Phi(0) = 0.5$, hence the distance between the policies is the same as in Proposition 2, i.e., as in the case where all districts are symmetric.

Using the fact that Φ is symmetric and hence $\phi'(0) = 0$ and $\Phi(0) = 0.5$, it is easy to verify that

$$\left.\frac{\partial(x_R-x_D)}{\partial M}\right|_{M=0}=0.$$

Hence, if $\left.\frac{\partial(x_R-x_D)}{\partial M}\right|_{M=0} < 0$, $M = 0$ is a local maximum, and polarization, i.e., the distance between the policies is smaller in a neighborhood of $M = 0$. The reverse is true if $\left.\frac{\partial(x_R-x_D)}{\partial M}\right|_{M=0} > 0$.

Again, using the fact that $\phi'(0) = 0$ and $\Phi(0) = 0.5$ it follows that

$$\left.\frac{\partial^2(x_R-x_D)}{\partial M^2}\right|_{M=0}=-\frac{4m^2\left(8\phi(0)^3+\phi''(0)+2m\phi''(0)\phi(0)\right)}{(1+2m\phi(0))^3}.$$

Thus, the second derivative is positive if and only if $8\phi(0)^3+\phi''(0)+2m\phi''(0)\phi(0) < 0$, i.e., if $m > \bar{m}$, where

$$\bar{m}=-\left(\frac{4\phi(0)^2}{\phi''(0)}+\frac{1}{2\phi(0)}\right).$$

Let $x_R^G(\bar{M})$, and $x_D^G(\bar{M})$ the policies in district $n+1$ in the gerrymandered model, and $x_R(\bar{M})$, $x_D(\bar{M})$ those in the symmetric model characterized in Proposition 2. Then for $\bar{M}$ in a neighborhood of zero, $\bar{M} \neq 0$ we get $x_R^G(\bar{M}) - x_D^G(\bar{M}) > x_R^G(0) - x_D^G(0)$, while item 3 in Proposition 2 implies $x_R(\bar{M}) - x_D(\bar{M}) < x_R(0) - x_D(0)$. Since $x_R^G(0) = x_R(0)$ and $x_D^G(0) = x_D(0)$ it follows that the equilibrium policies in district $n + 1$ are more polarized in the gerrymandered model for $m \geq \bar{m}$. ■

References

Adams, J. and S. Merrill (2003). Voter turnout and candidate strategies in american elections. *Journal of Politics 65*, 161–189.

Aldrich, J. H. (1995). *Why Parties?: The Origin and Transformation of Party Politics in America*. Cambridge Univ Press.

Banks, J. S. and J. Duggan (2005). Probabilistic voting in the spatial model of elections: The theory of office-motivated candidates. In *Social Choice and Strategic Decisions*, pp. 15–56. Springer: Berlin, Heidelberg.

Bernhardt, D., J. Duggan, and F. Squintani (2009). Private polling in elections and voter welfare. *Journal of Economic Theory 144*(5), 2021–2056.

Callander, S. (2005). Electoral competition in heterogeneous districts. *Journal of Political Economy 113*, 1116–1145.

Callander, S. (2008). Political motivations. *Review of Economic Studies 75*, 671–697.

Calvert, R. L. (1985). Robustness of the multidimensional voting model: Candidate motivations, uncertainty, and convergence. *American Journal of Political Science 29*, 69–95.

Camara, O. (2012). Economic policies of heterogeneous politicians. mimeo, USC.

Castanheira, M. (2003). Why vote for losers? *Journal of the European Economic Association 1*(5), 1207–1238.

Coleman, J. S. (1972). The positions of political parties in elections. In R. Niemi (Ed.), *Probability models of collective decision making*, pp. 332–57. Charles E. Merrill Columbus, OH.

Cox, G. W. and M. D. McCubbins (2005). *Setting the agenda: Responsible party government in the US House of Representatives*. Cambridge University Press.

Diermeier, D. and R. Vlaicu (2011). Parties, coalitions, and the internal organization of legislatures. *American Political Science Review 105*(2), 359–380.

Dixit, A. and J. Londregan (1995). Redistributive politics and economic efficiency. *American Political Science Review 89*(4), 856–866.

Downs, A. (1957). *An economic theory of democracy*. Harper.

Eguia, J. X. (2011a). Endogenous parties in an assembly. *American Journal of Political Science 55*, 16–26.

Eguia, J. X. (2011b). Voting blocs, party discipline and party formation. *Games and Economic Behavior 73*, 111–135.

Erikson, R. and D. Romero (1990). Candidate equilibrium and the behavioral model of the vote. *American Political Science Review 84*, 1103–1126.

Fiorina, M., S. Abrams, and J. Pope (2006). *Culture war? The myth of a polarized America*. Pearson Longman.

Gul, F. and W. Pesendorfer (2009). Partisan politics and aggregation failure with ignorant voters. *Journal of Economic Theory 144*, 146–174.

Hinich, M. (1978). The mean versus the median in spatial voting games. In P. Ordeshook (Ed.), *Game Theory and Political Science New York*. NYU Press: New York.

Hirano, S., J. M. Snyder Jr, S. Ansolabehere, and J. M. Hansen (2010). Primary elections and partisan polarization in the us congress. *Quarterly Journal of Political Science 5*(2), 169–191.

Krasa, S. and M. Polborn (2010a). The binary policy model. *Journal of Economic Theory 145*, 661–688.

Krasa, S. and M. Polborn (2010b). Competition between specialized candidates. *American Political Science Review 104*(4), 745–765.

Krasa, S. and M. Polborn (2012). Political competition between differentiated candidates. *Games and Economic Behavior 76*, 249–271.

Krasa, S. and M. Polborn (2013). Social ideology and taxes in a differentiated candidates framework. *American Economic Review*. forthcoming.

Lee, D. S., E. Moretti, and M. J. Butler (2004). Do voters affect or elect policies? Evidence from the US House. *The Quarterly Journal of Economics 119*(3), 807–859.

Lindbeck, A. and J. Weibull (1987). Balanced-budget redistribution as the outcome of political competition. *Public Choice 52*, 273–297.

Martinelli, C. (2001). Elections with privately informed parties and voters. *Public Choice 108*(1), 147–167.

McCarty, N., K. T. Poole, and H. Rosenthal (2009). Does gerrymandering cause polarization? *American Journal of Political Science 53*(3), 666–680.

Owen, G. and B. Grofman (2006). Two-stage electoral competition in two-party contests: persistent divergence of party positions. *Social Choice and Welfare 26*(3), 547–569.

Palfrey, T. R. (1984). Spatial equilibrium with entry. *Review of Economic Studies 51*, 139–156.

Patty, J. W. (2008). Equilibrium party government. *American Journal of Political Science 52*(3), 636–655.

Rohde, D. W. (2010). *Parties and leaders in the postreform House*. University of Chicago Press.

Soubeyran, R. (2009). Does a disadvantaged candidate choose an extremist position? *Annales d'Economie et de Statistique 93-94*, 328–348.

Volden, C. and E. Bergman (2006). How strong should our party be? party member preferences over party cohesion. *Legislative Studies Quarterly 31*(1), 71–104.

Wittman, D. (1983). Candidate motivation: A synthesis of alternative theories. *American Political Science Review 77*, 142–157.