The Political Intergenerational Welfare State: A Unified Framework

Monisankar Bishnu
Indian Statistical Institute, New Delhi
and Iowa State University

Min Wang
Peking University, Beijing

July 25, 2013

Abstract

This paper characterizes an intergenerational welfare state with education and pension under probabilistic voting where voters internalize the general equilibrium effects materializing in their life-span. We show that as public education is introduced in the economy through the political process, it always increases (reduces) the accumulation of human capital (physical capital), but strikingly, has no effect on the political equilibrium of pay-as-you-go (PAYG) social security tax. In fact, contrary to the popular belief, it reduces the generosity of pension benefits and therefore the idea of using public education as a weapon to handle the present social security issue stumbles. On the other hand, the introduction of a politically determined PAYG social security most definitely reduces physical capital accumulation, however its impact on human capital accumulation depends on how the education investment is financed. We also demonstrate that the general equilibrium effects are crucial to sustain the social security program, and explain why the presence of PAYG social security may not provide sufficient incentive for either public or private investment in education. Finally, we show that the simultaneous arrangement of public education and pension can increase the long-run growth if and only if the relative political influence of the old is small so that the pension program is thin.

Keywords: Education, Social security, Probabilistic voting, Markov Perfect Equilibrium, Endogenous growth

JEL Classification: E6, H3, H52, H55, D90

*Economics and Planning Unit, Indian Statistical Institute 7, S.J.S. Sansanwal Marg, New Delhi 110 016, India. Email: mbishnu@isid.ac.in.

†China Center for Economic Research, National School of Development, Peking University, Beijing 100871, China, Email: wangmin@nsd.edu.cn.
1 Introduction

Recently the concern of future solvency of social security program has invoked a heated public policy debate (see Cooley and Soares (1999), Boldrin and Rustichini (2000), Feldstein (2005) and Gonzalez-Eiras and Niepelt (2008) for example). While most of the papers in the literature concentrate only on pension itself, there are some studies (see for e.g., Pogue and Sgontz (1977), Becker and Murphy (1988) among the earlier works and more recently Rangel (2003), Boldrin and Montes (2005) among others) which emphasize that the other elements of the welfare state, particularly the forward channel, namely the education subsidy should not be ignored. Bringing both pension and education into the analysis not only makes the understanding of the functioning of welfare state more accurate and complete, but also reveals some of the more interesting features of the welfare state. Needless to say, the two-armed public program apparently looks more balanced since the working population on which the taxes are imposed are typically middle-aged and the two programs channelize the funds in two different directions. A question that is answered in this paper is how far do they balance each other.

Further, there are some important issues from the generational point of view that cannot be ignored when we analyze the two instruments of the welfare state. Among others, the first prominent issue that is frequently ignored is the intergenerational co-operation or conflict while allocating funds among different generations for different purposes. Indeed the generosity of these two programs, namely education subsidy and pension, is subject to an intergenerational co-operation or conflict. Naturally, the topic is interesting in a voting framework since these two programs are important elements of voting in any democratic economy. Given that this allocation is an outcome of voting procedure, the success of the wish (on the generosity of the program) of any generation depends on the distribution of voting power among the generations and this distribution has the ability to reflect the degree of co-operation or conflict among generations.\footnote{As argued by Mulligan and Sala-i-Martin (1999), the political clout of the old people has grown beyond the predicted by the evolution of the demographics. They interpret the data in favor of the old as a political power enjoyed by those citizen (also see Preston (1984), Lindert (1994)). As is mentioned in Song, Storesletten and Zilibotti (2012), the voter’s turnout in the US is falling however, the participation of the old is increasing. Further, they report that the share of vote by the old with 61+ age is expected to reach to 50\% by the year 2050 in the OECD countries. These phenomena well explain that in the voting framework, the equations are changing rapidly. Estimating a very small chance of success of the policy, these information may justify the concern why even with significant pressure on pension funds, the idea of lowering the tax rate has not been considered by the researchers.}

The second important thing that should not be forgotten is the general equilibrium effects of voting outcomes that voters take into account while voting. More precisely, when agents vote in our framework, they expect how the policy choice would affect the factor prices. This is however not present when the agents turn to the role of consumers. As we proceed, it will become evident that the general equilibrium effects not only are crucial to sustain the social security program, but also generate a clear distinction between the effects of public and private
investment in education. Further, they also explain why the presence of PAYG social security may not provide sufficient incentive for education investment, independent of its source. In this paper, thus, we focus on intergenerational conflict as well as the general equilibrium effects of any decision that originates from a voting outcome and show that both these factors have serious consequences on the states of the economy. Apart from other issues, the present study explores some of the interesting and timely relevant questions such as whether the existence of social security could increase the incentive to expand public investment in education, or as public education program provides more generous funding in education investment, whether the enhanced future productivity through augmenting education could help reduce the size of social security program.

The framework that we consider is a standard overlapping generations economy where altruistic agents live for three periods. We present a base line model which is a laissez-faire economy where education is privately provided through parental investment, and we then test it against different policy experiments step by step. Under the policy experiments, we first consider a situation where a public source of education is available but PAYG social security is absent. However this provision of public education is subject to voting on the issue of education tax. As the next policy experiment, we consider a political economy with a pension scheme in place; however the source of funding for education is now private. Thus the above two analyses deal with the economies where only one intergenerational transfer instrument is present. Finally, we integrate the two instruments together and consider an economy that not only has both the instruments, namely education subsidy and pension together, but also lets the agents vote simultaneously for these two instruments. Our dynamic political economy framework is a probabilistic voting model at each period where the voting power is distributed intergenerationally. In this equilibrium, voters are allowed to sequentially choose their policies under rational expectation about the effect on future outcomes. Further, we rely on the Markov Perfect Equilibrium (henceforth MPE) in the first two experiments and particularly on Simultaneous Nash MPE (which we call SNMPE and define later) for the last one. Our choice of probabilistic voting is driven by its simplicity and we depend on MPE for obtaining analytical results. But the additional gain from a probabilistic voting setup is that it can capture intergenerational co-operation and conflict, which is indeed the nature of competition between the two programs, namely education subsidy and pension. Here, the dynamic politico - economic equilibria are founded on competitive equilibrium with subgame perfect tax rates and transfers since the voters are not bound by their past political decisions. This complete analysis provides some important results that are worth noting and summarized below.

According to Pogue and Sgontz (1977), PAYG social security creates incentives for public investment in education. Becker and Murphy (1988) also suggest that PAYG social security

\[\text{For a discussion on MPE in overlapping generations framework, see Bhaskar (1998), Gonzalez-Eiras and Niepelt (2008) among others.}\]
strengthens the political support among the current working agents for public investment in education. Their study connects education investment made by the parents with pension by considering this as a trade among generations; children receive education from their parents and in exchange pay for their old age benefits. In a relatively recent work, Rangel (2003) also focuses on the issue of sustainability when both forward and backward intergenerational goods are present and shows that backward intergenerational goods (BIGs), such as social security, play a crucial role in sustaining investment in forward intergenerational goods (FIGs) like education: without them investment is inefficiently low, but with them optimal investment is possible. The first observation that we present in this paper is in contrast to these above partial equilibrium studies. We show that even with downward altruism, the introduction of PAYG social security may not provide sufficient incentive for public investment in education. However, this result is somewhat in line with Soares (2006) and Kaganovich and Zilcha (2012). Soares (2006) in a calibrated OLG framework finds that infusing a PAYG system results in a political equilibrium with lower funding for public education compared to the case when social security is absent. This happens because while PAYG scheme increases the voter’s incentive in investing young’s education to expand future tax base of pension program, it, via reducing the total savings, also increases the market interest rate, leading to a reduction of the present value of the pension benefits (Kaganovich and Zilcha, 2012). Thus our first observation guarantees that owning to the general equilibrium effects, the existence of PAYG may not increase the incentive for public education even when PAYG social security is endogenously determined. Further, when PAYG social security is present, we show how crucial the general equilibrium effects are to sustain the social security program, the channel which has also been emphasized by for e.g., Boldrin and Rustichini (2000) and Gonzalez-Eiras and Niepelt (2008).

Our second result conveys an important message that under MPE, the introduction of public education scheme by probabilistic voting in a laissez-faire economy increases human capital but reduces physical capital accumulation. This happens because voters take into account the positive general equilibrium effects of education investment and therefore would support a more generous funding for education compared to a situation when the funding for education is private. In addition to that, in the long run, when the production technology is more human capital intensive, it may increase the long run growth rate. Interestingly, this qualitative result is quite robust in nature in the sense that it holds even in the presence of PAYG social security. Additionally, we also find that when PAYG social security is already present, introducing public education has no effect on the equilibrium tax rate of PAYG social security, even though the public education program provides more generous funding in education than when education is privately funded. Taking a step further, we have been able to show that in fact due to the general equilibrium effects, the generosity of pension becomes even lower, despite the fact that the education level is higher than before. This finding has a strikingly strong implication. It implies that the existing wisdom that when PAYG pension is present, introducing a forward
intergenerational good, namely publicly funded education, can be helpful since it enhances future productivity, is no more correct. Keeping Gonzalez-Eiras and Niepelt (2008) in mind, if it is indeed needed to increase the pension tax rate in the future, we conjecture that even augmenting the level of education will not help in reducing the pension tax rate in a political equilibrium.

We, further, investigate the roles of PAYG social security in a political economy. We show that introducing PAYG social security with a positive pension tax in a laissez-faire economy reduces physical capital accumulation, but may increase education investment. However, if positive pension tax is introduced in an economy where publicly funded education is present, it will always lead to a reduction in both human and physical capital accumulation. The same interesting feature is reflected in terms of the tax rate too, that is, whenever the pension tax rate is positive, the PAYG social security will decrease the equilibrium tax rate of public education. This in fact guarantees that when publicly funded capital education is already present, introducing PAYG social security with a positive tax will hurt the long run growth.

As our final policy experiment, we introduce both public education and PAYG social security simultaneously in a laissez-faire economy. We find out that the simultaneous establishment of both the instruments always reduces the physical capital investment but human capital will increase if the relative weight to the old is small and thus the PAYG program is very thin. This is because in our model, the presence of PAYG discourages public investment in education and therefore the human capital production can be enhanced if and only if the size of the PAYG program is small as a result of the low relative weight assigned to the old retirees, which, in turn guarantees that the effect of this on public investment in education is very limited. Indeed the result raises an interesting point. In Boldrin and Montes (2005), designing simultaneous existence of public education and PAYG pension is justified as a means to implement an intergenerational transfer scheme that supports complete market allocations when credit market is missing. In our political economy setup, if the same question arises as to whether a simultaneous arrangement of these two-armed intergenerational transfers is justifiable for the long run growth, the answer would depend on the distribution of political power among the generations and thus making the Boldrin and Montes (2005) result “conditional” in our framework.\footnote{Wang (2013) extends their study by endogenizing the imperfection of the credit market and shows that the result could hold even when borrowing constraints for education loan arise endogenously. In another interesting work, Andersen and Bhattacharya (2013) show that when the young must borrow to make the education investment and the economy is dynamically efficient, the golden rule level of human capital is higher than that is achievable with complete education-loan markets alone. Further, they show that a carefully designed education-pension welfare package can achieve the golden rule level of human capital, and owning to the intergenerational human capital externality, the pension component of such a package will be entirely phased out eventually.}

\footnote{According to Docquier, Paddison and Pestieau (2007), the definition of optimality in Boldrin and Montes (2005) framework is restrictive where the study disregards the effect of externality in education. They show that because of the externalities, allocations of human and physical in competitive equilibrium differ from the planner’s and a possibility naturally arises where the laissez-faire equilibrium experiences higher physical capital}
The present study can be seen as a political economy version of Rangel (2003) with two-armed intergenerational transfers. An important feature besides providing a political structure is the consideration of the general equilibrium effect, which turns out be crucial as mentioned above and is missing from Rangel (2003). The present paper can also be seen from an angle where Gonzalez-Eiras and Niepelt (2008) appears as one of the special cases in which human capital and therefore one of the two (intergenerational) arms of the welfare state, namely education, is truncated from the analysis. Naturally, in this light, the present paper seems more complete and has the flexibility to compare all the situations, specifically both private and public channels of investment in education and PAYG social security. We here must mention some other papers that also study education and social security in unison and thus are related to our study. Kaganovich and Zilcha (1999) consider an economy with altruistically-motivated parents who invest in the human capital of their children, and analyze how the allocation of the tax revenues between public education and social security affects growth and welfare. In their study, however, the total tax rate is artificially fixed and the allocation of tax revenue is determined by a growth maximization problem. Pecchenino and Utendorf (1999) study the impact of PAYG social security on incentives for private investment in education and shows that in an aging economy, social pension may crowd out education investment and thus reduces growth and social welfare. Poutvaara (2006) uses trigger strategies to characterize the political equilibrium of pension and public education in an economy where agents are heterogeneous and decisions are taken by majority voting. In an open economy framework with fixed factor prices, Naito (2012) also focuses on the sustainability of public education and pension under probabilistic voting setup and investigates the interaction between these two policies with economic growth. However in his study, the proportion of expenditure on one program to the total expenditure is artificially fixed at some level. Kaganovich and Zilcha (2012) study a political economy in which public education is determined through a voting process but the social security tax rate is exogenously given. They show that compared to PAYG, the fully funded social security system produces political support for a relatively higher education funding, and hence generates higher accumulation but lower human capital accumulation compared to the planner’s allocations. However, Bishnu (2013) shows that if the origin of non-optimality of human capital accumulation is the consumption externality, the possibility that the accumulation of human and physical capital in a laissez-faire differ from the planner’s in opposite direction is not at all feasible. This observation not only has crucial implication on pension and education subsidy but also can justify government intervention in education even in the absence of education externalities.

The paper that is technically close to ours is Gonzalez-Eiras and Niepelt (2008) which deals with a probabilistic voting model (only on pension) with population growth in which they take into account the general equilibrium effects of the existing policy on the future outcome. Though we do not consider any population growth, the essence of changing population is captured through the distribution of intergenerational voting power. Using a similar framework as in Gonzalez-Eiras and Niepelt (2008), Song (2011) focuses on within generation heterogeneity and analyzes the interaction between social security transfers and wealth inequality. He finds that higher inequality is associated with higher equilibrium social security tax rates if social security redistributes within cohorts. This paper too relies on probabilistic voting and uses MPE, however like Gonzalez-Eiras and Niepelt (2008), ignores the education channel of transfers.
rates of physical and human capital accumulation and economic growth. In a very recent study, Ono (2013) incorporates longevity as well as altruism and allows voting on both pension and publicly funded education. Though the framework and the focus are different from ours, on the technical front, this is the only paper beside ours which considers voting on bi-dimensional policy issues (as in subsection 3.3 below). Some of the earlier studies that considered the link between public education and public pension are Richman and Stagner (1986), Cremer, Kessler and Pestieau (1992) among others.

Thus by comparing the effects of the two different sources of funding for education (both separately and in unison) along with PAYG in a political economy framework, where both the intergenerational conflict and the general equilibrium effects are considered, the present paper certainly distinguishes itself from the existing literature. The study not only provides a comprehensive analysis which was somehow ignored in the literature, it also clearly suggests that a partial framework that ignores the crucial effects that mentioned above may produce different results. Further, on the technical ground, to the best of our knowledge, this model is the first attempt to study simultaneous voting on the two instruments (tax rates) of intergenerational transfers in a probabilistic voting model.

The rest of the paper is organized as follows. Section 2 presents the baseline model, that is, the laissez-faire economy. In section 3, we present the three policy experiments where different instruments are politically determined through a probabilistic voting model. Section 4 deals with the welfare implication of the intergenerational transfers. While section 5 concludes, proofs are presented in the Appendix.

2 Laissez-faire Economy

We consider an economy that consists of an infinite sequence of three-period lived overlapping generations, an initial old generation and an initial middle-aged generation. In each generation, there is a continuum of identical agents of measure one. Agents receive education when young, while they work and carry out decisions of consumption, saving and education investment for their children during middle age. When they are old, the agents retire and consume out of the total return on their savings. An agent who is working at period $t$ that is a middle-aged agent is called a generation-$t$ agent.

Denote by $h_t$ the human capital of an individual belonging to generation $t$. The human capital of a generation $t + 1$ agent is a function of educational expenditure $e_t$ she makes when young and her parent’s human capital $h_t$, the endowment of basic knowledge she is born with. We assume the human capital is produced by a constant-return-to-scale technology $h_{t+1} = h(e_t, h_t) = Be_t \beta h_t^{1-\beta}$, $B > 0$, $\beta \in (0, 1)$.

Most of our results hold for a human capital specification that abstracts from generational human capital externalities, that is, where $h_{t+1}$ depends on $e_t$ only.
returns to scale production function \( F(K_t, H_t) \), where \( K_t \) and \( H_t \) are aggregate physical capital and human capital at \( t \). Defining \( k_t \equiv K_t / N_t \) and \( h_t \equiv H_t / N_t \) in which \( N_t \) is the population of generation-\( t \) agents, output per middle-aged agent at time \( t \) can be expressed as an intensive form \( f(k_t, h_t) = F(k_t, h_t) \). We assume that \( f \) takes the Cobb-Douglas form, i.e., \( f(k_t, h_t) = A k_t^\alpha h_t^{1-\alpha}, A > 0, \alpha \in (0,1) \). Since the measure of the members of each generation is one, we know \( N_t = 1 \) and thus \( k_t = K_t \) and \( h_t = H_t \). The final good can either be consumed in the period it is produced, or it can be saved to provide capital in the following period. Capital is conveniently assumed to depreciate fully between periods. Young agents supply labor inelastically in competitive labor markets, earning a wage of \( w_t = \partial f(k_t, h_t) / \partial h_t = (1 - \alpha) A k_t^\alpha h_t^{-\alpha} \) at time \( t \); similarly, capital is traded in competitive capital markets, and earns a gross real return of \( R_{t+1} \) between \( t \) and \( t+1 \) where \( R_t = \partial f(k_t, h_t) / \partial k_t = \alpha A k_t^{\alpha-1} h_t^{1-\alpha} \).

We assume a generation-\( t \) agent draws utility from \((c_t^m, c_{t+1}^o, h_{t+1})\), the terms in the parenthesis denoting consumption at middle age and old age, and the level of human capital of her children respectively. More specifically, the life-time utility for a generation-\( t \) agent is

\[
    U \equiv u(c_t^m) + \delta \left[ u(c_{t+1}^o) + \phi u(h_{t+1}) \right] \tag{1}
\]

where \( \delta \in (0,1) \) is the standard discount factor and \( \phi \in (0,1) \) represents the relative weight assigned to the utility that an old agent enjoys from her children’s human capital, expressing parents’ altruism towards their offspring.\(^7\) Instantaneous utility function \( u \) is continuously differentiable, strictly increasing and strictly concave. An agent when middle-aged allocates her labor income among consumption, saving and investment in education for her children.\(^8\) Saving \( s_t \) while middle-aged returns \( s_t R_{t+1} \) in the next period when the agent is old. These imply that \( c_t^m = w_t h_t - s_t - e_t \) and \( c_{t+1}^o = s_t R_{t+1} \). Thus given the factor prices, human capital production technology \( h_t = h(e_{t-1}, h_{t-1}) \) and the budget constraints stated above, when a generation-\( t \) agent maximizes her utility (1) with respect to \( \{s_t, e_t\} \), we arrive at the following first order

\(^7\)Utility specification that represents altruism through the level of human capital of the next generation is very common and vastly used in the literature. Our specification is simple and standard, for example, as in Kaganovich and Zilcha (1999), Pecchenino and Utendorf (1999) Glomm and Kaganovich (2003, 2008) and in line with Glomm and Ravikumar (1992), De la Croix and Doepke (2003) among others.

\(^8\)As a common treatment in the literature, we, in this paper, do not bring a credit market that can fund education for the young. All the discussions here are on the two sources, private investment (made by parents) and public investment (made by the government), due to the following considerations. First, these two sources are the main sources of funds in any economy. Second, owning to the inalienability of human capital, future labor income cannot be collateralized and credit markets severely restrict any borrowing against future human capital for education purposes. A credit market for education loan even does not exist in most developing countries, e.g., China or trivially thin as in India. Third, the present setup allows us to derive analytical results.
conditions:

\[ u'(w_t h_t - s^L_t - e^L_t) = \delta R_{t+1} u'(R_{t+1}s^L_t) \]  
(2)

\[ u'(w_t h_t - s^L_t - e^L_t) = \delta \phi u'(h_{t+1}) \frac{\partial h(e^L_t, h_t)}{\partial e^L_t}. \]  
(3)

While equation (2) represents the optimal intertemporal consumption allocation, that is the standard Euler’s equation, (3) indicates that the marginal sacrifice in utility from investing in education of the descendents is equal to the marginal benefit from utility gain adjusted to the gain in the level of human capital of their descendents. To derive closed-form solutions of political equilibrium and economic growth, we must impose functional form restrictions on the utility function (1). Specifically, we assume that the utility function is logarithmic. Specifically by solving the above two first order conditions, we get the followings:

\[ e^L_t = \frac{\beta \delta \phi}{1 + \delta + \beta \delta \phi} w_t h_t \]  
(4)

\[ s^L_t = \frac{\delta}{1 + \delta + \beta \delta \phi} w_t h_t. \]  
(5)

Using the above along with the equilibrium factor prices and the fact that the general-equilibrium condition \( k_{t+1} = s_t \) holds at every \( t \), we obtain a two-dimensional first-order dynamical system of this economy, specifically, \( k^L_{t+1} = \delta (1 - \alpha) A k_t^{\alpha} h_t^{1-\alpha} / (1 + \delta + \beta \delta \phi) \) and \( h^L_{t+1} = B [\beta \delta \phi A (1 - \alpha) / (1 + \delta + \beta \delta \phi)]^{1/\beta} k_t^{\alpha \beta} h_t^{1-\alpha \beta} \). Given \( k_0 \) and \( h_0 \), all the dynamic competitive equilibria are characterized by the sequences of \( \{k^L_t, h^L_t\} \) that satisfy the two equilibrium paths expressed above. Given this dynamical system, we complete our characterization for this laissez-faire economy by focusing at the steady state equilibrium and thus we have the following lemma:

**Lemma 1** *In the laissez-faire economy, there exists an unique steady state with balanced growth where the human and physical capital grow at a same constant rate.*

### 3 Political Economy

In this section we consider the same framework as before but additionally introduce two oppositely directed policies, namely public education and social security sequentially to investigate the roles that they play in an economy. We bring these two intergenerational welfare states

---

Superscript \( L \) represents the optimal and equilibrium values of the concerned variables in the laissez-faire economy. Similarly, the superscripts \( G, P \) and \( X \) are used to represent the optimal and equilibrium values of the concerned variables in the political economy of public education, political economy of pension and political economy of two-armed intergenerational transfers analysis as in subsections 3.1, 3.2, and 3.3 respectively.
through a political process which runs under a probabilistic voting framework (Lindbeck and Weibull, 1987), and the economy politically determines the size of these programs to be implemented.

In particular, we investigate three political economies. The choices are naturally not arbitrary, rather it helps providing a complete understanding of all possible scenarios. The first scenario is the Political economy of public education presented in subsection 3.1. In this economy, the only political decision made by the agents is determining the level of public investment in education. That is, the government collects tax revenue from the middle-aged agents and channelizes it to the present young to provide funds for their education. The second scenario is the Political economy of pension which is presented in subsection 3.2. This economy deals with another instrument of welfare state but it is a backward intergenerational good, namely social security. However, in this case, we keep the source of education funding private as in the laissez-faire economy, but introduce public pension benefit in the form of politically determined PAYG social security. The last two scenarios bring out the effects of the instruments when they are present in an economy in isolation. Thus, as a last policy experiment, in the Political economy of two-armed intergenerational transfers, we bring both of the two instruments together through a political process that simultaneously determines these two intergenerational goods. With these three policy environments and the laissez-faire economy presented above, we not only can explore the political equilibrium of public education and PAYG social security, but can also examine their roles in all possible scenarios. In particular we can investigate the political equilibrium and the roles of public education or PAYG social security when the other good is existent and nonexistent respectively.

3.1 Political Economy of Public Education

We first consider a modification of the laissez-faire economy, where social security is absent but the agents politically determine the size of the public education program. Suppose the government imposes a proportional tax rate $\theta_t$ at each $t$ on the income earned by each of the generation $- t$ agents when they are middle-aged to finance the public education program for the present young. The fiscal program of subsidy needs to satisfy the period-wise balanced budget condition, i.e., $e_t = \theta_t w_t h_t$. The tax rate $\theta_t$ is determined by a repeated political process (to be discussed below in detail) at the beginning of each period. After the political process of voting is complete and the education tax rate is set, agents make their decisions on consumption and savings. Given the factor prices, education policies and the human capital production technology, a generation-$t$ agent’s optimization problem now is to maximize (1) subject to $c^m_t = (1 - \theta_t) w_t h_t - s_t$ and $c^s_{t+1} = s_t R_{t+1}$. Then given $\theta_t$, the first order condition
with respect to \( s_t \) is given by

\[
u' \left[ (1 - \theta_t) w_t h_t - s_t^G \right] = \delta R_{t+1} u' \left( R_{t+1} s_t^G \right)\]

which ensures the optimal saving

\[
s_t^G = \frac{(1 - \theta_t) \delta w_t h_t}{1 + \delta}. \tag{6}\]

Next, we solve the political equilibrium under a repeated voting process where only middle-aged and old participate. Abiding by the standard practice as discussed earlier, we disallow the young’s participation in the voting process due to age-restriction. At the beginning of each period, the contemporaneous tax rate is determined by a candidate who is democratically elected by all the current voters. We assume that the size of the program is determined in a probabilistic-voting framework. Under this political setting, there are two political candidates who are competing in an election. When deciding on which candidate to support, voters anticipate the effects of the candidate’s policy platform on equilibrium prices, future’s political decisions and their own welfare. As we mentioned earlier we focus on the MPE where agents can form perfect foresight on the policies which depend on the set of state variables of the economy. For rest of the analysis, we use the notation \( S^t \) to denote the set of state variables in period \( t \), i.e., \( S^t \equiv \{ k_t, h_t \} \). Since winning the election is the only aim of the candidates, in the probabilistic-voting Nash-equilibrium, the two candidates seeking to maximize their vote shares propose the same policy platform. This policy platform maximizes a weighted average of the welfare of all voters, in which the weights assigned to different groups of voters reflect the size or the political power of different generations.

By the foregoing discussion, the political decision on the equilibrium education policy can be derived by an exercise that maximizes the weighted sum of the indirect utilities of two generations, i.e.,

\[
\max_{\theta_t \in [0,1]} W \left( S^t; \theta_t \right) = \omega V_t^o \left( S^t; \theta_t \right) + V_t^m \left( S^t; \theta_t \right), \tag{7}\]

where \( V_t^m \) and \( V_t^o \) respectively denote the welfare of the middle aged and the old at period \( t \) given the state \( S^t \). The parameter \( \omega \) is the political weight assigned to the old relative to a middle-aged by the political candidates. In line with the explanation by Song, Storesletten and Zilibotti (2012), this relative weight captures the relative political clout of each generation, reflecting, on one hand, its relative size and on the other hand, exogenous group-specific characteristics, such as the voting turnout or the salience of the fiscal policy for that group relative to other issues. In our model, the young receive education, but (7) shows that this particular generation is not allowed to vote and thus is completely dependent on others for investment in education. It should be noticed that although the young have no role in the voting process, the middle-aged have an incentive to invest in education for the future generation since they directly derive
utility out of the level of education accumulated by their descendents. Not only that, agents also could acquire higher returns on their saving in the next period through general equilibrium effects.

To characterize the political decision on the public education investment, we first consider the welfare effect of this equilibrium education tax $\theta_t$ on various groups of voters, i.e., the middle-aged workers and the old retirees. Evidently, the education tax $\theta_t$ imposed in period $t$ has no welfare effect on the current old, i.e., $\partial V_o^t/\partial \theta_t = 0$. This follows directly from the fact that all the variables, i.e., $h_t$, $R_t$ and $s_{t-1}$, in the utility function of the old in period $t$, $V_o^t = u(R_t s_{t-1}) + \phi u(h_t)$, are pre-determined in $t$. For the middle aged, the welfare effect of education tax is more complex. Differentiating $V_m^t$ with respect to $\theta_t$ yields

$$
\frac{\partial V_m^t}{\partial \theta_t} = -u'(c^m) w_t h_t + \delta \phi u'(h_{t+1}) \frac{\partial h_{t+1}}{\partial \theta_t} + \delta s_{t+1}^G u'(c^m) \left( \frac{\partial R_{t+1}}{\partial h_{t+1}} + \frac{\partial s_{t+1}^G}{\partial \theta_t} \right) \left( \frac{\partial h_{t+1}}{\partial \theta_t} \right) . \quad (8)
$$

Note that the effect of $\theta_t$ on the savings of the middle aged cancels out by the envelope theorem. The first negative term $A$ reflects the cost of investment in public education. The second term $B$ captures the positive effect of public education through tax enjoyed by the parental generation because of altruism. As afore-discussed, the last two terms $C$ and $D$ reflect the general equilibrium effect of public education tax on the rate of interest through the channel of physical and human capital respectively. On one hand, by directing funds as a forward intergenerational good to the next generation, the education tax $\theta_t$ reduces private savings in period $t$ and consequently also reduces the physical capital investment, which eventually leads to an increase in the rate of interest in the next period. That is, $\partial R_{t+1}/\partial k_{t+1} < 0$ along with $\partial k_{t+1}/\partial \theta_t < 0$. On the other hand, channelizing more funds towards the education of the next generation necessarily increases the level of human capital of the descendents, which in turn, also increases the rate of interest, i.e., $\partial R_{t+1}/\partial h_{t+1} > 0$ along with $\partial h_{t+1}/\partial \theta_t > 0$. The aggregate general equilibrium effect of investment in public education is thus positive for the middle-aged workers with $C > 0$ and $D > 0$.

Under MPE, the equilibrium tax rate is the fixed point in $\theta_t(S^t) = \arg \max_{\theta_t \in [0,1]} W(S^t; \theta_t)$. We first substitute the factor prices, the equilibrium condition for market clearing and the private optimal savings $s^G_t$ given by (6) into $W(S^t; \theta_t)$. Then, by omitting the terms independent of policy parameter $\theta_t$, the political objective function $W(S^t; \theta_t)$ reduces to

$$
W(S^t; \theta_t) \simeq (1 + \alpha \delta) \ln(1 - \theta_t) + \beta \delta (1 - \alpha + \phi) \ln \theta_t.
$$

\[10\] Note that under our specific functional form of the human capital production, $\partial h_{t+1}/\partial \theta_t = B\beta c_i^\beta - 1 h_i^{\beta - 1} w_i h_t$. 

\[11\] In all that follows, we will use the notation $\simeq$ to denote the effective value function that contains the relevant fiscal parameter but not the other irrelevant terms.
By solving the first order condition of the above probabilistic-voting problem, i.e., by setting \( \partial W (S^t, \theta_t) / \partial \theta_t = 0 \), we finally have the following lemma:

**Lemma 2** In a political economy with public education determined in a probabilistic-voting setting, there exists an unique interior education tax rate \( \theta^G \) under MPE and is given by

\[
\theta^G \equiv \frac{\phi \beta \delta + \beta \delta (1 - \alpha)}{\Omega} \in (0, 1) \forall t
\]

where \( \Omega = 1 + \alpha \delta + \beta \delta (1 - \alpha) + \beta \delta \phi > 1 \).

Lemma 2 shows that, under the logarithm utility, the equilibrium education tax is independent of the states of the economy and thus is constant over time. Using the equilibrium conditions and the balanced budget program of the government, the two-dimensional first-order dynamical system of this economy can be written as

\[
k_{t+1}^G = A \left[ \delta A (1 - \theta^G) (1 - \alpha) / (1 + \delta) \right] k_t^G h_t^{1 - \alpha} + \beta k_t^G h_t^{1 - \alpha}.
\]

Given \( k_0 \) and \( h_0 \), all dynamic, competitive equilibria are characterized by sequences of \( \{k_t^G, h_t^G\} \) that satisfy the above two equilibrium paths.\(^{12}\)

Finally we must mention that although we exclude from here private investment in education, relaxing this assumption, i.e., allowing parents to invest in children’s education, will not change the results. The following corollary indeed shows that private investment in education cannot coexist with publicly provided education that is made available through the political process of voting.

**Corollary 1** In the political economy of public education, private investment in education is optimally driven to the zero corner when public education is politically determined.

### 3.2 Political Economy of Pension

We now proceed to consider an economy that politically implements only a social security program. Just as in a laissez-faire economy discussed in section 2, a proportional tax \( \tau_t \) is imposed at period \( t \) on the wage income earned by a generation–\( t \) agent. The total tax revenue is then collected and used up to provide pay-as-you-go social security \( b_t \) to the old generation at \( t \). The government’s budget balance requirement for this program ensures \( b_t = \tau_t w_t h_t \). The social security tax rate, \( \tau_t \), is determined in the same political setting as in section 3.1. Given the factor prices, human capital production technology, the social security program and the budget constraints, a generation–\( t \) agent’s optimization problem now involves maximizing (1)
subject to $c_t^m = (1 - \tau_t) w_t h_t - s_t - e_t$ and $c_{t+1}^o = R_{t+1} s_t + b_{t+1}$. Thus given $\tau_t$, the first order conditions with respect to $(e_t, s_t)$ are given by

$$u' \left[ (1 - \tau_t) w_t h_t - s_t^P - e_t^P \right] = \delta \phi u' \left( h_{t+1} \right) \frac{\partial h_{t+1}}{\partial e_t}$$

and

$$u' \left[ (1 - \tau_t) w_t h_t - s_t^P - e_t^P \right] = \delta R_{t+1} u' \left( R_{t+1} s_t^P + b_{t+1} \right).$$

Solving the above two simultaneously results in the following equilibrium values of $e_t^P$ and $s_t^P$:

$$e_t^P = \beta \delta \phi \left( 1 - \tau_t \right) \frac{w_t h_t + b_{t+1}/R_{t+1}}{1 + \delta + \beta \delta \phi} \tag{9}$$

and

$$s_t^P = \left( 1 - \tau_t \right) \frac{w_t h_t - (1 + \beta \delta \phi) b_{t+1}/R_{t+1}}{1 + \delta + \beta \delta \phi}. \tag{10}$$

As in section 3.1, we next consider the welfare effects of the social security tax $\tau_t$ on the middle-aged workers and the old retirees. Differentiating the utility of the old with respect to $\tau_t$ yields $\partial V_o^t / \partial \tau_t = u' (c_t^o) w_t h_t > 0$. Since the old benefit from the social security program without bearing any cost, it is evident that they always prefer a tax rate that is as high as possible. Compared to the political economy of public education, in which the utility of the present old is not at all affected by the tax rate on education, either directly or through the general equilibrium effect, here in the presence of backward intergenerational goods, e.g., PAYG social security, the old will have a role to play in the political decision making process.

By using the government’s balanced budget condition $b_t = \tau_t w_t h_t$, market clearing condition $k_{t+1} = s_t$, and the factor prices, the welfare effect of pension tax $\tau_t$ on a generation—$t$ agent is

$$\frac{\partial V_t^m}{\partial \tau_t} = -u' (c_t^m) w_t h_t + \delta u' \left( c_{t+1}^o \right) \left( \tau_{t+1} w_{t+1} \frac{\partial h_{t+1}}{\partial \tau_t} + w_{t+1} h_{t+1} \frac{\partial \tau_{t+1}}{\partial \tau_t} + \mathcal{H} \right) \tag{11}$$

where

$$\mathcal{H} = s_t^P \left( \frac{\partial R_{t+1}}{\partial k_{t+1}} \frac{\partial \tau_{t+1}}{\partial \tau_t} + \frac{\partial R_{t+1}}{\partial h_{t+1}} \frac{\partial \tau_{t+1}}{\partial \tau_t} \right) + \tau_{t+1} h_{t+1} \left( \frac{\partial w_{t+1}}{\partial k_{t+1}} \frac{\partial \tau_{t+1}}{\partial \tau_t} + \frac{\partial w_{t+1}}{\partial h_{t+1}} \frac{\partial \tau_{t+1}}{\partial \tau_t} \right).$$

The first negative term $\mathcal{E}$ reflects the direct cost of social security contributions. The second term $\mathcal{F}$ captures the effect of social security tax which reduces the level of education of the

---

\(^{13}\)By the envelope theorem, the effect of $\tau_t^P$ on the savings and education investment of the middle aged cancels out.
next generation and consequently the transfer benefits that the present middle aged generation receives when old. Explicitly, this effect of social security tax on the level of education acquired by the next generation can be derived from the expression of equilibrium $e_t^P$ (equation (9)) and also by using the equilibrium prices as follows:

$$\frac{\partial h_{t+1}}{\partial \tau_t} = \frac{\beta h_{t+1}}{(1 - \tau_t) w_t h_t + b_{t+1}/R_{t+1}} \left\{ -w_t h_t + \frac{(1 - \alpha)}{\alpha} \frac{\partial}{\partial \tau_t} \left( \tau_{t+1} k_{t+1} \right) \right\}. \quad (12)$$

According to (9), the social security affects the human capital accumulation by changing the present value of life-time income $(1 - \tau_t) w_t h_t + b_{t+1}/R_{t+1}$ through two different channels: 1) by directly reducing the wage income of the middle aged, it crowds out parental investment in children’s education and 2) by indirectly changing the equilibrium factor prices and the future social security program, it changes the present value of future social security benefit $b_{t+1}/R_{t+1}$. Notice that in equilibrium, $b_{t+1}/R_{t+1} = \tau_{t+1} w_{t+1} h_{t+1}/R_{t+1} = (1 - \alpha) \tau_{t+1} k_{t+1}/\alpha$. Further, as will be shown below, in equilibrium $\tau_{t+1}$ is independent of $\tau_t$. This together with the fact that $\partial k_{t+1}/\partial \tau_t < 0$, results in the negativity of the second indirect effect. That is increasing current social security tax rate would reduce the present value of future social security benefit for the current middle aged. As a result $I$ as well as $\partial h_{t+1}/\partial \tau_t$ is negative. The third term $G$ captures the effect of the current tax choice on the future tax outcome and will be cancelled out if the equilibrium tax rate is independent of the states of the economy.

The last term $H$ reflects the general equilibrium effect of social security tax through the channel of physical and human capital on the factor prices. By shifting income from the middle-aged to the old, the PAYG pension reduces savings as well as the physical capital investment. The effect of pension tax on the physical capital investment is thus negative i.e., $\partial k_{t+1}/\partial \tau_t < 0$. Here the movements of capital in terms of backward intergenerational goods not only decreases the level of physical capital in the future, thus increasing the rate of interest, but also decreases the level of investment in human capital for the next generation, consequently reducing human capital in the future. This in turn leads to an opposite effect (compared to physical capital) on the rate of interest. We see a similar thing happening with the other factor price, namely wages. While wages fall owing to the fact that there is a loss in physical capital due to backward transfer, we will also expect the price of human capital to be augmented since human capital also goes down in the future. Thus, unlike the previous case of political economy of public education, there is always a tension between the two opposite effects of two types of capital on the factor prices. Therefore $H^1$ and $H^4$ are positive and $H^2$ and $H^3$ are negative, with the result that the sign of the aggregate general equilibrium effect of the social security tax, $H$, is ambiguous. However since $E$ and $F$ are negative, if the social security can be maintained in equilibrium, the aggregate general equilibrium effect of the social security tax must be positive. Indeed this particular result has support from the existing literature. As emphasized by, for e.g.,
Boldrin and Rustichini (2000) and Gonzalez-Eiras and Niepelt (2008), the general equilibrium effect is very crucial in sustaining the social security program in equilibrium. Our analysis in a different framework, in fact, reconfirms the necessity of the general equilibrium effect that keeps PAYG program sustainable.

In equilibrium, as usual, political candidates who maximize their respective vote shares propose the same policy platform and maximize the combination of the welfare of all voters, which is given by \( W(S^t; \tau_t) = \omega V_t^o(S^t; \tau_t) + V_t^m(S^t; \tau_t) \). First by substituting the factor prices, private optimal savings and education investment (expressions (10), (9)), and imposing the equilibrium condition \( k_{t+1} = s_t \) in \( W(S^t; \tau_t) \), and then by omitting the terms independent of policy parameter \( \tau_t \), the political objective function \( W(S^t; \tau_t) \) reduces to

\[
W(S^t; \tau_t) \simeq \omega \ln[\alpha + \tau_t (1 - \alpha)] + \Omega \ln(1 - \tau_t).
\]

Following Gonzalez-Eiras and Niepelt (2008), we here make a conjecture that future equilibrium policy \( \tau_{t+1} \) is independent of current political choice of \( \tau_t \), which will be verified to be indeed the case. Solving the first order condition of the probabilistic-voting problem, i.e., by setting \( \partial W(S^t; \tau_t) / \partial \tau_t = 0 \), we have

**Lemma 3** In a political economy with PAYG pension determined in a probabilistic-voting setting, there exists an unique social security tax rate \( \tau^P \) under MPE and is given by

\[
\tau^P = \frac{\omega - \alpha \Omega / (1 - \alpha)}{\omega + \Omega} < 1 \quad \forall t.
\]

Under logarithmic utility, the equilibrium social security tax is independent of the states of the economy, verifying the conjecture we made above. Notice that while the tax rate on education in the previous case of political economy of public education is bounded between zero and unity, in case of pension tax there is no guarantee that it will always be non-negative.\(^{14}\) By using the government’s balanced budget condition \( b_t = \tau_t w_t h_t \), and the market clearing condition \( k_{t+1} = s_t \), the two-dimensional first-order dynamical system of the economy can be written as \( k_{t+1}^P = \alpha \delta A k_t^P h_t^{1-\alpha} / [\alpha \delta + \omega (1 + \alpha \delta + \beta \delta \phi) / \Omega] \) and \( h_{t+1}^P = B k_t^\alpha h_t^{1+\beta} [A \omega \beta \delta \phi / [(\alpha \delta + \omega (1 + \alpha \delta + \beta \delta \phi) / \Omega) (\omega + \Omega)]}^\beta \). Given \( k_0 \) and \( h_0 \), all dynamic, competitive equilibria are characterized by sequences of \( \{k_t^P, h_t^P\} \) that satisfy the two equilibrium paths as expressed above.

\(^{14}\) A point to note here is that our analysis can generate the results in Gonzalez-Eiras and Niepelt (2008) as a special case. Gonzalez-Eiras and Niepelt (2008) uses a two-period overlapping generations model that features a positive population growth \( n_t \) in order to study the pension tax. If we impose \( \beta = 0 \) to shut out the education investment in our model, the equilibrium pension tax is \( \tau^P = [\omega - \alpha (1 + \alpha \delta) / (1 - \alpha)] / (\omega + 1 + \alpha \delta) \), which is exactly equal to the result of Gonzalez-Eiras and Niepelt (2008) with \( n_t = 1 \).
Political Economy of Two-Armed Intergenerational Transfers

While in the previous two subsections we deal with only one fiscal instrument at a time, this subsection deals with the situation when both forward and backward intergenerational goods are present, that is, when the economy is two-armed. To the best of our knowledge, this is the first attempt towards studying simultaneous political decisions on two intergenerational tax instruments in a general equilibrium setup. In this setting, government has two public programs and the budget balance conditions require \( e_t = \theta_t w_t h_t \) and \( b_t = \tau_t w_t h_t \) where \( \theta_t \) and \( \tau_t \) are the education and pension tax rate respectively imposed on the wage income of the middle-aged workers. Given the tax rates, an agent maximizes (1) subject to

\[
ct = (1 - \theta_t - \tau_t) w_t h_t - s_t
\]

and

\[
c_{t+1} = R_{t+1} s_t + b_{t+1} .^{15}
\]

Solving the generation \( t \) agent’s optimization problem, we obtain

\[
s_t^* = \frac{(1 - \theta_t - \tau_t) \delta w_t h_t - b_{t+1}/R_{t+1}}{1 + \delta} . \quad (13)
\]

We will now consider the welfare effects of social security tax and education tax on the middle-aged workers and the old retirees. It can be verified that for the old, (given \( \tau_t \)) the welfare effect of education tax, i.e., \( \partial V^o_t / \partial \theta_t = 0 \), and (given \( \theta_t \)) the welfare effect of social security tax, i.e., \( \partial V^o_t / \partial \tau_t = u'(c^o_t) w_t h_t \), remain same as in the previous cases. Since neither of the two public policies can modify the current states of the economy, they can have only the direct effect of taxation on the old. Further, it can be verified that (given \( \theta_t \)) the welfare effect of social security tax on the middle-aged is the same as in the case (see equation 11) in which only social security policy is present; however, (given \( \tau_t \)) the welfare effect of education tax changes to

\[
\frac{\partial V^m_t}{\partial \theta_t} = J + \delta u'(c^o_{t+1}) \left[ \frac{\partial (\tau_{t+1} w_{t+1} h_{t+1})}{\partial \theta_t} \right] K \quad \frac{\partial (\tau_{t+1} w_{t+1} h_{t+1})}{\partial \theta_t} \quad (14)
\]

where \( J \) denotes the right hand side of (8). Further, when a pension tax accompanies an education tax, the term \( K \) represents an extra welfare effect of education tax on the middle-aged where

\[
\frac{\partial (\tau_{t+1} w_{t+1} h_{t+1})}{\partial \theta_t} = \tau_{t+1} w_{t+1} h_{t+1} \frac{\partial h_{t+1}}{\partial \theta_t} + \tau_{t+1} w_{t+1} h_{t+1} \left( \frac{\partial w_{t+1}}{\partial k_{t+1}} \frac{\partial k_{t+1}}{\partial \theta_t} + \frac{\partial w_{t+1}}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial \theta_t} \right) + w_{t+1} h_{t+1} \frac{\partial \tau_{t+1}}{\partial \theta_t} . \quad (15)
\]

As we can see \( K \) captures the welfare effect of education tax that the agents pay in the middle age on the social security benefit that they will receive in the future when old. As discussed previously, the general equilibrium effects of public education are \( \partial k_{t+1} / \partial \theta_t < 0 \) and \( \partial h_{t+1} / \partial \theta_t > 0 \), along with \( \partial w_{t+1} / \partial k_{t+1} > 0 \) and \( \partial w_{t+1} / \partial h_{t+1} < 0 \). Given the sign of these effects along with

---

15 In order to make the model consistent with that in subsection 3.1, we here also exclude the private investment in education from agent’s choice set.
the fact that $\partial \tau_{t+1} / \partial \theta_t = 0$ (as will be shown below), the sign of $K$ becomes indeterminate.

Note that in the economy where only education policy is in existence and is politically determined, the middle aged agents support the education tax because they can gain from the investment in public education through parental altruism as well as from the positive general equilibrium effect of education tax on the interest rate. Our analysis confirms that in the presence of pension, the effect of education tax is not necessarily positive. The result explains that introducing PAYG may not increase the benefits from public education monotonically. This is in contrast to the existing results that appear in Pogue and Sgontz (1977), Becker and Murphy (1988) and Rangel (2003), where the authors based their study in a partial equilibrium framework. They argue that since the enhanced human capital of the next generation can increase the future social security benefit, the presence of social security would increase the incentive of the middle-aged in investing in the public education. In fact, in (15), if we drop all the terms that are related to general equilibrium effects, that is, except the first term, we can verify that an increase in education will have beneficial effect on the generosity of pension when old, given $\partial h_{t+1} / \partial \theta_t > 0$. This is in the same spirit as the key result of the above three papers mentioned above. However, the result changes once the general equilibrium effects are taken into account and this somewhat supports the findings by Soares (2006) and very recently by Kaganovich and Zilcha (2012).

In equilibrium, the objective function for the political candidates becomes $W(S^t; \theta_t, \tau_t) = \omega V^o_t (S^t; \theta_t, \tau_t) + V^{m}_t (S^t; \theta_t, \tau_t)$. By substituting agent’s optimal savings (13), government’s balanced budgets for the two programs, factor prices and equilibrium market clearing condition into $W(S^t; \theta_t, \tau_t)$, and then dropping the irrelevant terms, the political objective function $W(S^t; \theta_t, \tau_t)$ reduces to

$$W(S^t; \theta_t, \tau_t) \approx \omega \ln [\alpha + \tau_t (1 - \alpha)] + (1 + \alpha \delta) \ln (1 - \theta_t - \tau_t) + \beta \delta (1 - \alpha + \phi) \ln (\theta_t).$$

The optimal tax rates are simultaneously determined in a repeated probabilistic voting. We solve for the two tax rates by solving the two first order conditions $\partial W(S^t; \theta_t, \tau_t) / \partial \theta_t = 0$ and $\partial W(S^t; \theta_t, \tau_t) / \partial \tau_t = 0$ simultaneously. We term this equilibrium as Simultaneous Nash Markov Perfect Equilibrium (SNMPE). As before, we make the conjecture that the future equilibrium policy $\tau_{t+1}$ is independent of current political choice of $\theta_t$ and $\tau_t$ and will verify that it is indeed the case. Solving the SNMPE for this probabilistic-voting problem gives us the following lemma.

**Lemma 4** In a political economy with public education and PAYG pension determined in a probabilistic-voting setting, there exists a unique set of instruments $(\theta^X, \tau^X)$ under SNMPE where $\tau^X = \tau^P$ and

$$\theta^X = \frac{\beta \delta (1 - \alpha + \phi) / (1 - \alpha)}{\omega + \Omega} \in (0, 1) \ \forall t.$$
Under logarithmic utility, the equilibrium social security tax is independent of the states of the economy, verifying the conjecture. Under this repeated stage game, every subgame thus has an unique Nash equilibrium. This in fact confirms that this is the unique Nash equilibrium for the entire game. Thus the short run taxes that have been solved simultaneously at each period $t$, are valid not only for each $t$ but also valid for the long run. Using these two equilibrium tax rates, the equilibrium market clearing condition and the budget constraints, the two-dimensional first-order dynamical system of the economy can be defined by $k_{t+1}^X = \alpha \delta (1 + \alpha \delta) A k_t^\alpha h_t^{1-\alpha}/[\omega + \alpha \delta (\omega + \Omega)]$ and $h_{t+1}^X = B [A \beta \delta (1 - \alpha + \phi)/ (\omega + \Omega)]^\beta k_t^{\alpha \beta} h_t^{1-\alpha \beta}$. Given $k_0$ and $h_0$, all dynamic, competitive equilibria are characterized by sequences of $\{k_t^X, h_t^X\}$ that satisfy the above two equilibrium paths. Finally we complete the whole section by presenting the steady state equilibrium for the above three political economies.

**Lemma 5** In each of the three political economies presented above, there exists a unique steady state with balanced growth where the human and physical capital grow at a same constant rate.

### 4 Welfare Implications of Political Intergenerational Transfers

As two of the largest public transfer programs in most economies, public education and social security evoke tremendous research interest in the academia and have been intensively studied in the literature. Both public education and social security are commonly viewed as a mechanism of intergenerational transfers, but as mentioned in the introduction, there is a wide diversity of studies on these two transfers but most of them are in isolation. We present a unified framework for studying intergenerational transfer in a general equilibrium political economy model. The results could be misleading if we ignore the existence of the other policy due to the possible interaction between the two intergenerational transfers in a political economy. This unified framework allows us to compare\(^{16}\) the results of different treatments of the two intergenerational transfers that have been partially conducted in previous studies. Furthermore, within our framework, we can investigate the difference in the role played by one of the two intergenerational transfers when the other transfer running in the opposite direction is either absent or present.

#### 4.1 Public Education

As is the standard practice, we first examine the role of public education in a political economy in the absence of PAYG social security program. In all that follows, we particularly concentrate

\(^{16}\)Whenever a comparison is made in our analysis, we naturally assume that the state variables at period $t$ are same for all the cases. Changes appear from $t+1$ and the crucial variables are represented with the superscripts then.
on the welfare effects of the intergenerational transfer policy on the accumulation of both human and physical capital in the short run as well as their long-run growth. Comparison of laissez-faire equilibrium with the equilibrium of the political economy of public education leads to the following proposition.

**Proposition 1** Under MPE, the introduction of public education in a laissez-faire economy by the process of probabilistic voting

1) at each \( t \), given \( S^t \), increases the human capital accumulation and reduces physical capital accumulation. Further it

2) increases the long-run growth rate when \( \alpha \) is small.

The above proposition ensures that while on one hand, investment in education is higher when education is publicly provided (compared to when it is not, as in a laissez faire economy), an economy with public education generates lower amount of physical capital in the short run.\(^{17}\) The first part of the proposition conveys an interesting message that the arrangement of public education is more generous than private in terms of the level of education produced in an economy and interestingly, the result holds independent of the intergenerational distribution of weights. Since implementation of a system of public education has no impact on the current old retirees when pension is absent, evidently it is the middle aged that decide to increase education investment in the political process. As a voter, when deciding on investment in education, besides the altruism effect, the middle-aged agent also takes into account the positive general equilibrium effect of education investment which she is unaware of as a consumer. As a consequence, the education investment under the political process is higher than the private education investment. This increase in education investment in turn crowds out savings, therefore reducing the physical capital investment. If we shut out the general equilibrium effect of education investment in the voting process, i.e., \( C \) and \( D \) in equation (8), the public education investment would be exactly equal to the private education investment. Although the public education generates countervailing effects on the accumulation of physical capital and human capital, if the production function is human capital intensive, then the human capital enhancement effect dominates so that public education ensures higher growth in the long run.

We proceed to investigate whether the role of public education changes when the PAYG social security has been present in the economy. Following the same line of comparison as above, it can be verified that the presence of PAYG social security does not change the qualitative welfare effects of public education. The positive general equilibrium effects of education are thus attained in the new political economy. That is introducing public education generates

\(^{17}\)There are some existing studies where private and public regimes are compared. One influential study, namely Glomm and Ravikumar (1992), shows that if schooling time is also an input in human capital production and the young need to allocate time between schooling and leisure, private education investment is higher than public education investment in every time period. Our finding is however the opposite.
the same results as in Proposition 1 even when the PAYG social security has already been politically determined in the economy. This is stated below as Proposition 2.

**Proposition 2** The results of Proposition 1 hold in a political economy with PAYG social security.

In addition to exploring the welfare analysis of an intergenerational transfer policy in different scenarios, in this paper we are also interested in how the introduction of an intergenerational transfer policy affects the other transfer policy that is already present in the economy. The next proposition follows directly from Lemma 4.

**Proposition 3** When PAYG social security is already present in the economy, introducing politically determined public education has no effect on the political equilibrium tax rate of PAYG social security, but affects the generosity of pension adversely.

When PAYG is already present, bringing in education tax does not affect the welfare effect of social security tax on the present old, but would influence the middle aged via direct effect of taxation as well as through the general equilibrium effects. For the middle-aged, it is a replacement of private education, $e_t^P$, by the public education investment, $\theta_t^X w_t h_t$, and therefore they are forced to pay more for the education of the next generation. This has a direct consequences through the change in the level of education of their descendents and not only that, the individual decision (of savings) and equilibrium market prices also change under such a replacement. However, Proposition 3 surprisingly shows that the social security tax rate on the middle aged remains unchanged in spite of the change in the levels of accumulation of both physical and human capital. In fact our observation is even stronger. Since the presence of public funds in education reduces the capital stock (though increases the level of human capital) as shown above, given $b_{t+1}/R_{t+1} = (1 - \alpha) k_{t+1} \tau_{t+1}/\alpha$ holds at the equilibrium, the present value of pension in fact decreases.

This result is striking and has important implications and is also in line with what we observe in the economy. First it shows that when there is a change in the amount of public fund for education, there is rarely a change in the tax rate of PAYG social security. The stronger implication of the result is that the reduction in social security tax rate can easily be ruled out in a voting equilibrium. People should not expect that government would increase the public education investment by sacrificing the benefit of PAYG social security for the current old. A notable example is that when U.S. government had to face a tight budget during the recent financial crisis, it had shut down many elementary and middle schools and cut down funding support for public universities, but it never thought of reducing the benefit of PAYG social security.

Second, it has been argued by many authors that by suitably extending the forwarding arm, backward arm pension can be balancing and stable. An implied view is that by increasing
the level of education so that the future income is expanded, the burden of social security tax may be reduced. However, we have been able to show that, although the public education program provides more funding for education investment in the next generation than parental investment, if PAYG social pension is politically determined, the pension tax rate would be unchanged by the political decision on public education. In fact, the increase in the level of education does not at all guarantee an increase in the pension benefit. Thus our result has a very clear implication. Gonzalez-Eiras and Niepelt (2008) as well as the social security administration have projected that the social security tax rate should increase it in the future in response to demographic change. We show that even after adopting the remedial measure of augmenting the level of education, there might be immense difficulty in bringing about any change in tax rate in a democratic setup.

In sum, in this subsection we have shown that although the quantity of the welfare effects may differ, the qualitative welfare effects of public education on capital accumulation and long-run growth remains the same irrespective of whether the PAYG social security is present or not. Further, it turns out that where the tax rate is concerned, social security program would not be affected by the political decision on public education even when the two instruments are generationally well linked.

4.2 PAYG Social Security

Just as in the previous subsection, we first consider the role of PAYG social security in a political economy that stays away from public education. To do that, we will examine the welfare effects of PAYG social security by comparing the laissez-faire economy with a political economy that is accompanied by private education and chooses PAYG social security through the voting process. After that, following the same exercise, we will study the role of PAYG social security in an economy where public funding in education that has been decided through a political process is present. Finally, we discuss how PAYG social security affects the political decision on public education.

We start with focusing on the role of PAYG social security in the case of private education and present our first proposition below.

**Proposition 4** Under MPE, there exist two threshold weights

\[
\tilde{\omega} = \frac{\alpha \Omega}{1 - \alpha} \quad \text{and} \quad \tilde{\omega} = \frac{\delta (1 - \alpha) \Omega}{1 + \alpha \delta + \beta \delta \phi}
\]

such that introducing PAYG social security by probabilistic voting in a laissez-faire economy, at each \( t \), given \( S^t \),

1) increases the physical capital accumulation if and only if \( \omega \leq \tilde{\omega} \).
2) and also given \( \bar{\omega} \leq \tilde{\omega} \) or \( \tilde{\omega} \geq \bar{\omega} \), increases the human capital accumulation if and only if 
\[ \omega \in [\bar{\omega}, \tilde{\omega}] \quad \text{or} \quad \omega \in [\tilde{\omega}, \bar{\omega}], \]
where \( \tilde{\omega} \geq \bar{\omega} \) is equivalent to \( \alpha (1 + \beta \delta \phi + 2 \delta) \geq \delta \).

Note that the political equilibrium of social security tax \( \tau^P \) is increasing in the political power of the old retirees \( \omega \). In addition, it can easily be verified that \( \tilde{\omega} \) is the value at which \( \tau^P \) equals to zero. Hence the equilibrium social security tax is positive if and only if \( \omega \geq \tilde{\omega} \), and consequently, as is standard in the literature, positive (negative) social security tax decreases (increases) total savings as well as physical capital investment of the economy, from which the first statement of Proposition 4 follows directly.\(^{18}\)

Proposition 4 also demonstrates that, depending on the relative political weight of the old retirees, introducing the PAYG social security program could increase or decrease human capital investment, and the relationship between the political weight and the welfare effect of PAYG social security on human capital investment is non-monotonic. This happens because as the political weight of the old retirees rises, both the current and the future social security tax would rise, which according to (9), would further affect the human capital investment via different ways by changing the present lifecycle income. Precisely,

\[
\frac{\partial e^P_t}{\partial \omega} = \frac{\beta \delta \phi}{1 + \delta + \beta \delta \phi} \left[ T \bigg|_{\tau_1 = \tau_{t+1} = \tau^P} \frac{\partial \tau^P}{\partial \omega} + \left( \frac{1 - \alpha}{\alpha} k_{t+1}^P \frac{\partial \tau^P}{\partial \omega} \right) \right].
\]

The term \( T \bigg|_{\tau_1 = \tau_{t+1} = \tau^P} \) captures the effect of increasing current social security tax rate on the present life-time income, which as discussed in subsection (3.2) is negative when the social security tax rate is positive. If the social security tax rate is negative, then \( T \bigg|_{\tau_1 = \tau_{t+1} = \tau^P} \) could either be negative with a smaller absolute value or positive due to the fact that \( \partial k_{t+1}/\partial \tau^P > 0 \).

On the other hand, since in equilibrium \( b_{t+1}/R_{t+1} = (1 - \alpha) k_{t+1}^P/\alpha \), \( \mathcal{L} \) represents the positive effect of an increase in future social security tax on the present lifecycle income. Thus the increase of the political weight of the old retirees could generate two countervailing effects on the education investment. As shown in the proof for Proposition 4, \( e^P_t \) actually is concave in \( \omega \). When \( \omega \) is sufficiently small so that social security tax is negative, the effect \( T \bigg|_{\tau_1 = \tau_{t+1} = \tau^P} \) is limited as discussed previously and \( \mathcal{L} \) is large due to enhanced physical capital investment \( k_{t+1}^P \). In this case, the welfare effect of \( \omega \) on education investment is dominated by the above mentioned positive effect, and education investment is positively related to the political weight of the old. The opposite is true when \( \omega \) is sufficiently large so that the negative effect dominates and the education investment becomes decreasing in the political weight of the old. On the other hand, the human capital investment in the laissez-faire economy is independent of the political weight. Hence introducing the PAYG social security program could increase the human capital investment if and only if the old have intermediate-level political power.

\(^{18}\)Since \( \omega \) is the key to the intergenerational conflicts or co-operation, we use the parameter \( \omega \) when comparing.
Note that according to Proposition 4, if $\bar{\omega} \leq \omega$, there could exists a range of $\omega$, i.e., $\omega \in [\bar{\omega}, \tilde{\omega}]$, in which bringing the PAYG social security into an economy that education investment source is private could increase both physical capital investment and human capital investment. In this case, the PAYG social security tax is negative, but it undoubtedly enhances the long run growth rate. However, if the political weight is beyond that range, $\omega \notin [\bar{\omega}, \tilde{\omega}]$ or $\omega \notin [\bar{\omega}, \tilde{\omega}]$, the social security discourages both physical capital investment and human capital investment, thereby hurting the long run growth. Another interesting result that one needs to note is that, if $\bar{\omega} \leq \omega$ and $\omega \in [\tilde{\omega}, \tilde{\omega}]$, the politically determined PAYG social security program could increase the parental investment in education even when the social security tax is positive. In this case, the social security has countervailing effect on both physical and human capital investment.

Now we focus on the economy where education is publicly provided and compare the situation in which PAYG social security is present with the one where it is not. Interestingly, when the source of education is public, although the welfare effect of PAYG social security on physical capital investment is the same as stated in the above proposition, its welfare effect on human capital investment differs radically.

**Proposition 5** Consider a political economy where the source of education is public. Introducing PAYG social security that is politically determined through a voting process would increase both physical and human capital investment if and only if $\omega < \bar{\omega}$.

Since the politically decided social security tax rate is same irrespective of whether education is privately or publicly funded, the welfare effect of social security on physical capital investment is same as before and thus straightforward.

We have learned from Proposition 4 that when expenditure on education is a private decision as in a laissez-faire economy, introducing politically determined PAYG social security increases the education investment if and only if the political power of the old is at intermediate level. However, here when education investment is determined by a voting process, introducing politically determined PAYG social pension would increase the education investment if and only if the political power of the old is so small that the social security tax is negative. Before the PAYG social security is injected into the political economy of public education, the old have no role in the political decision on public education. Thus the education tax is independent of the political weight $\omega$. When the PAYG social security program is introduced in the economy, the public education tax rate $\theta^p$ becomes monotonically decreasing in the political weight of the old. The reason behind this phenomenon is that, when PAYG is introduced, it would compete with the public education program that is already present for government revenue. High political weight of the old yields high social security tax rate, which in turn would crowd out the public education investment. When $\omega$ is in the range of $\omega \geq \tilde{\omega}$ so that the pension tax rate is indeed positive, bringing PAYG social security in an economy where public education is present always reduces the public education investment as well as the tax rate.
Since now the welfare effect of PAYG social security on the physical and human capital goes in the same direction, the effect of PAYG social pension on the long run growth rate is straightforward. That is, introducing politically determined PAYG social pension into a political economy with public education would increase the long run growth rate if and only if $\omega < \bar{\omega}$, which is totally different from the previous case where public education is absent. If we further restrict $\omega$ to be above $\bar{\omega}$, that is if, $\omega \geq \bar{\omega}$ so that the PAYG social security is positive, then PAYG social security always hurts the long-run growth rate.

The subsection 4.1 has shown that when PAYG social pension is already present in the economy, introducing politically determined public education into the economy results in no change in the PAYG social security tax rate. Here, directly following Proposition 5, we see a different picture. That is, when public education is already present in the economy, introducing politically determined PAYG social security would increase the equilibrium tax rate of public education if and only if the political power of the old is small enough, i.e., $\omega \leq \bar{\omega}$. Earlier studies, like Pogue and Sgontz (1977) and Rangel (2003), argue that the presence of PAYG social security would increase the middle aged’ support for public education, because they will be rewarded with larger social security benefit when they are old owning to the enhanced human capital of the next generation. However, Soares (2006) and Kaganovich and Zilcha (2012) have shown that this is not necessarily true in a general equilibrium model, because the positive effect of increased education investment on future social security benefit due to the increased tax base would be cancelled out by the negative general equilibrium effect of education investment on factor prices. Interestingly, their result can be obtained in our study even if we allow the PAYG social security program to be endogenously determined and the sources of funding in education is made private.

Since public education has been existent in the history for several hundred years, and the PAYG social security came into the picture only in the twentieth century, the result has a very important implication for the real economies. It suggests that the public education investment would be generally hurt if an economy decides to implement PAYG social security through a voting process. On the other hand, in terms of recent discussion on reforming the current PAYG social security program, our result implies that dismantling or reducing the size of the current PAYG social security program could help increase the public education investment and therefore benefit the long run growth rate.

### 4.3 Fully Fiscal

In previous subsections, we investigated the role of public education or PAYG social security by sequentially introducing the two intergenerational transfer policies into an economy one by one. In this subsection, we deal with the extreme case where PAYG social pension and public education are simultaneously brought to a laissez-faire economy by a voting process, and
examine their aggregate welfare effects on the economy. Comparison of laissez-faire equilibrium with the political equilibrium of the economy with public education and PAYG social security simultaneously determined gives

**Proposition 6** In a political economy under MPE, simultaneous determination of subsidy on education and social security through the political process of voting would

1) increase the capital investment if and only if $\omega \leq \bar{\omega}_1$, and

2) increase the education investment if and only if $\omega \leq \bar{\omega}_2$, where $\bar{\omega}_2 > \bar{\omega} > \bar{\omega}_1$.

Proposition 6 shows that, if an economy decides to simultaneously build up public education program and PAYG social security program by a political process, there exists three regimes of the welfare states: a) when $\omega \in [0, \bar{\omega}_1]$, the investment of both physical and human capital increase; b) when $\omega \in [\bar{\omega}_1, \bar{\omega}_2]$, where physical capital investment decreases while the human capital investment still increases; c) when $\omega \in [\bar{\omega}_2, \infty)$, the investment of both physical and human capital decrease. If we restrict $\omega$ in the range of $\omega \geq \bar{\omega}$ so that the PAYG social security is positive, the simultaneous determination of public education and PAYG social security would always reduce the physical capital investment, but would increase the human capital investment if and only if the political weight of the old is relatively small so that the size of PAYG social security program is thin, i.e., $\omega \in [\bar{\omega}, \bar{\omega}_2]$. This result is intuitive: we have shown in Proposition 1 and 2 that as public education is introduced to an economy, it always increases the human capital investment no matter whether the PAYG social security is present or not, but here due to the presence of PAYG social security, which, as shown in Proposition 5, would discourage the public education investment. The human capital investment thus can be enhanced (compared to the laissez-faire private human capital investment) if and only if the PAYG social security is small so that its effect on public education investment is limited.

In Boldrin and Montes (2005), prescribing simultaneous establishment of public education and PAYG social security is justified by their role in restoring the complete market solution when credit market is imperfect. In our political economy, if we check the welfare effects of the two intergenerational transfer policies on the long-run growth, whether simultaneous establishment of these two-armed intergenerational transfer are justifiable depends on the distribution of the political power. If the political weight of the old is relatively low so that the PAYG social security tax is negative, i.e., $\omega \leq \bar{\omega}_1$, building up public education and PAYG social security simultaneously in a political economy would increase investment in both the human and physical capital, thereby increasing the long-run growth rate. However, if we consider the more realistic case that the political weight of the old is relatively large so that the PAYG social security tax is positive, then simultaneous subsidy on education and social security would definitely hurt the long-run growth of the economy when $\omega \geq \bar{\omega}_2$, and may or may not hurt the long-run growth of the economy when $\omega \in [\bar{\omega}, \bar{\omega}_2]$. In the latter case, human capital investment is enhanced, but the physical capital investment falls. As discussed above, in such a case, only when the production
function is human capital intensive, i.e., $\alpha$ is small, then the human capital enhancement effect dominates so that the long-run growth rate can be enhanced. In sum, in our political model, the rationale for simultaneous establishment of standard pattern of intergenerational transfers (the PAYG social security is positive) is pretty limited.

5 Conclusion

Education (a FIG) and pension (a BIG) are the two biggest sources of spending by any welfare state. Though the sources of funding for education and pension in an economy are two widely discussed topics, a complete and comprehensive study of these intergenerational goods in a unified framework was missing from the literature. This paper tries to fill in this gap through a political economy approach. More particularly, the paper relies on probabilistic voting which can also be seen as a benevolent government’s problem with particular weights being assigned to different generations. The solving technique involves the idea of a MPE and for some results we employ the notion of SNMPE. We consider an economy where altruism is observed and middle-aged and old agents vote to choose their own tax rates. Because of the age restriction, young cannot participate in voting. The intergenerational instruments we consider are education subsidy and pension. We introduce these instruments in a sequential manner in a laissez-faire economy. We also study the case where both these instruments exist at the same time in the economy. The laissez-faire economy here is one in which no intergenerational instruments are present. Thus young agents depend fully on parental investment for their education. We assume that the agents care about the (general equilibrium) effect of the present policies on the future factor prices and other future crucial variables that are materialized in their life-time.

Given this framework, our study reveals many important results which make our understanding of the effects of these intergenerational transfer policies clear. We show that when public education is introduced in an economy through a political process of voting, it always reduces the accumulation of physical capital but increases the accumulation of human capital. However, it has no effect on the PAYG social security tax rate. A stronger result that emerges is that even with an increase in the level of education under the public regime, the generosity of pension actually falls due to the erosion of physical capital stock. This is in contrast to the popular wisdom that changing the public funds available for education may reduce the tax rate on PAYG social security. On the other hand, we have been able to show that introducing PAYG social security that has been politically determined, most definitely reduces physical capital accumulation. But human capital will be reduced with certainty if only if the public education is already present in the economy. Otherwise, human capital actually increases. Further, we demonstrate that the general equilibrium effects are crucial in sustaining the social security program, and explain why the presence of social security may not provide sufficient
incentive for investment in education. This result is robust since it holds independent of the source of education. Finally, when we introduce both public education and positive PAYG in a laissez-faire economy, we find that the presence of both the instruments always reduces the physical capital investment but increases human capital if the relative political weight to the old is small and therefore the PAYG program is thin. If the question arises whether simultaneous arrangement of these two-armed intergenerational transfers are justifiable for an increase in the long run growth, the answer in our model would depend on the distribution of political power and thus making the result of Boldrin and Montes (2005) study "conditional" in our political economy setup.

A nice extension to this study would be to accommodate fertility and longevity since we cannot ignore the effects of these in our framework, especially when the intergenerational equations are changing rapidly. We leave this for our future study.
References


6 Appendix A

Proof of Lemma 1. First, using the expressions for the paths of capital accumulation, we have
\[
\frac{k_{t+1}^L}{h_{t+1}^L} = \left( \frac{[A(1-\alpha)]^{1-\beta}}{B(1+\delta+\beta\delta\phi)^{1-\beta} (\beta\phi)^\beta} \right) \left( \frac{k_t^L}{h_t^L} \right)^{\alpha-\alpha\beta}.
\]
Then by denoting \( \bar{k}_{t+1}^L \equiv k_{t+1}^L/h_{t+1}^L \), we can observe that \( \bar{k}_{t+1}^L \) is concave in \( k_t^L \) with \( \lim_{k_t^L \to 0} \bar{k}_{t+1}^L = 0 \) and \( \lim_{k_t^L \to 0} d\bar{k}_{t+1}^L/dk_t^L = \infty \). Hence there exists a unique non-trivial steady state value of \( \bar{k}^L \) which is given by
\[
\bar{k}^L = \left\{ \frac{[A(1-\alpha)]^{1-\beta}}{B(1+\delta+\beta\delta\phi)^{1-\beta} (\beta\phi)^\beta} \right\}^{\frac{1}{1-\alpha+\alpha\beta}}.
\]
Further, at the steady state, the growth rate of the human capital is given by
\[
\frac{h_{t+1}^L}{h_t^L} = B \left[ \frac{\beta\delta\phi A(1-\alpha)}{1+\delta+\beta\delta\phi} \right]^{\beta} \left( \bar{k}^L \right)^{\alpha\beta} = \left\{ \frac{B^{1-\alpha} (\beta\phi)^{\beta(1-\alpha)} [A(1-\alpha)]^{\beta}}{(1+\delta+\beta\delta\phi)^{\beta}} \right\}^{\frac{1}{1-\alpha+\alpha\beta}},
\]
which is a constant. It is straightforward to verify that this is also the steady state growth rate for physical capital. Hence the proof.

Proof of Lemma 2. Note that \( \theta_t^G \) is the solution to the first order condition, \( \partial W(S_t^t; \theta_t) / \partial \theta_t = 0 \). It is to be noted that \( \theta_t^G = \theta_t^G \forall t \). Further, we can verify \( \lim_{\theta_t \to 0} W(S_t^t; \theta_t) = \lim_{\theta_t \to 1} W(S_t^t; \theta_t) = -\infty \), and \( \partial^2 W(S_t^t; \theta_t) / \partial \theta_t^2 < 0 \) so that the second order sufficient condition is satisfied. Hence the proof.

Proof of Corollary 1. When both the sources of education investment are present, agents maximize (1) subject to \( c_t^a = (1-\theta) w_t h_t - d_t - s_t \), \( c_t^{t+1} = R_{t+1} s_t \), \( e_t = d_t + g_t \) and \( d_t \geq 0 \), where \( \theta_t \) is the education tax rate, \( d_t \) and \( g_t \) are the new notations we use for private and public investment in education (we in fact use these notations for this proof only). With the assumption of logarithmic utility and government’s budget balance, \( g_t = \theta_t w_t h_t \), we have
\[
d_t^* = \begin{cases} \frac{\beta\delta\phi - \theta_t(1+\delta+\beta\delta\phi)}{1+\delta+\beta\delta\phi} w_t h_t, & \text{if } \theta_t < \frac{\beta\delta\phi}{1+\delta+\beta\delta\phi} \\ 0, & \text{if } \theta_t \geq \frac{\beta\delta\phi}{1+\delta+\beta\delta\phi} \end{cases}
\]
and
\[
s_t^* = \begin{cases} \frac{\delta w_t h_t}{1+\delta+\beta\delta\phi}, & \text{if } \theta_t < \frac{\beta\delta\phi}{1+\delta+\beta\delta\phi} \\ \frac{\delta(1-\theta_t) w_t h_t}{1+\delta}, & \text{if } \theta_t \geq \frac{\beta\delta\phi}{1+\delta+\beta\delta\phi}. \end{cases}
\]
Next we solve the political equilibrium of education tax rate. First, when \( \theta_t < \beta\delta\phi / (1+\delta+\beta\delta\phi) \),
that is when \( c_i^* > 0 \), we have

\[
W(S^i; \theta_i) \approx [\beta \delta (1 - \alpha) + \beta \delta \phi] \ln [\beta \delta \phi - \theta_i (\delta + \beta \delta \phi)].
\]

The maximum occurs when \( \theta_i = 0 \), which guarantees that \( d_i^* > 0 \) along with \( \theta_i > 0 \) cannot co-exist. Secondly, when \( \theta_i \geq \beta \delta \phi / (1 + \delta + \beta \delta \phi) \), that is when \( d_i^* = 0 \), as shown in Lemma 2, the political equilibrium of education tax rate is \( \theta_i \equiv \theta^G \). We can verify that \( \theta^G > \beta \delta \phi / (1 + \delta + \beta \delta \phi) \) holds since this is equivalent to \( (1 - \alpha) + \delta [1 + \alpha (1 - \phi)] > 0 \) which is true. ■

**Proof of Lemma 3.** It is easy to check that \( \tau_i^G \) is the solution to \( \partial W(S^i; \tau_i) / \partial \tau_i = 0 \). It can also be verified that \( \partial^2 W(S^i; \tau_i) / \partial \tau_i^2 < 0 \) with \( \lim_{\tau_i \to 0} W(S^i; \tau_i) = \omega \ln \alpha \) and \( \lim_{\tau_i \to 1} W(S; \tau_i) = -\infty \). Hence the proof. ■

**Proof of Lemma 4.** Since education subsidy and pension tax are simultaneously chosen under a Nash setting, one tax rate is determined assuming the other tax rate is given at the optimum level. Thus, we find out the optimal tax rates \( (\theta_i^X, \tau_i^X) \) by simultaneously solving the two first order conditions as follows:

\[
\begin{align*}
\frac{\partial W(S^i; \theta_i, \tau_i)}{\partial \tau_i} &= \frac{\omega (1 - \alpha)}{\alpha + \tau_i^X (1 - \alpha)} - \frac{1 + \alpha \delta}{1 - \tau_i^X - \theta_i^X} = 0, \\
\frac{\partial W(S^i; \theta_i, \tau_i)}{\partial \theta_i} &= \frac{\beta \delta (1 - \alpha + \phi)}{\theta_i^X} - \frac{1 + \alpha \delta}{1 - \tau_i^X - \theta_i^X} = 0.
\end{align*}
\]

It can also be verified that the Hessian of \( W(S^i; \theta_i, \tau_i) \),

\[
\begin{pmatrix}
-\frac{\omega (1 - \alpha)^2}{[\alpha + \tau_i^X (1 - \alpha)]^2} & -\frac{1 + \alpha \delta}{(1 - \theta_i^X - \tau_i^X)^2} \\
-\frac{1 + \alpha \delta}{(1 - \theta_i^X - \tau_i^X)^2} & -\frac{\beta \delta (1 - \alpha + \phi)}{(\theta_i^X)^2}
\end{pmatrix}
\]

is negative definite so that the second order sufficient condition is satisfied. Hence the proof. ■

**Proof of Lemma 5.** This proof follows the same approach of proof of Lemma 1. First we can show that for each political economy, there exists a unique non-trivial steady state value of \( \kappa^i \equiv k^i/h^i \), \( i = \{L, P, X\} \), which are respectively given by

\[
\begin{align*}
\kappa^G &= \left\{ \frac{\delta (1 - \theta^G) (1 - \alpha)}{B (1 + \delta)} \left[ \theta^G (1 - \alpha) A \right]^{\frac{1}{1 - \alpha + \alpha \delta}} \right\}, \\
\kappa^P &= \left\{ \frac{\alpha \delta A (1 - \alpha) (1 - \tau^P) / \left\{ \beta \delta \phi A (1 - \tau^P) [\alpha + (1 - \alpha) \tau^P] \right\}^{\beta}}{B \left\{ \alpha (1 + \delta + \beta \delta \phi) + (1 - \alpha) (1 + \beta \delta \phi) \tau^P \right\}^{1 - \beta}} \right\}^{\frac{1}{1 - \alpha + \alpha \delta}}, \\
\kappa^X &= \left\{ \frac{\frac{1}{B \left[ \omega + \alpha \delta (\omega + \Omega) \right] A \beta (1 - \alpha + \phi) / (\omega + \Omega) \right\}^{\beta}}{1 - \alpha + \alpha \delta} \right\}.
\end{align*}
\]
Further, we can show that for each political economy, at the steady state, both the human capital and physical capital grow at a constant rate and are respectively given by

\[
\frac{h_{G t+1}^G}{h_t^G} = B \left[ \theta^G A \left( 1 - \alpha \right) \right] \left( \frac{\tau^G}{\kappa} \right)^{\alpha \beta},
\]

\[
\frac{h_{P t+1}^P}{h_t^P} = B \left\{ \frac{\delta \beta A \left( 1 - \tau^P \right) \left[ \alpha + (1 - \alpha) \tau^P \right]}{\alpha (1 + \delta + \delta \beta \phi) / (1 - \alpha) + (1 + \beta \delta \phi) \tau^P} \right\} \left( \frac{\tau^P}{\kappa} \right)^{\alpha \beta},
\]

\[
\frac{h_{X t+1}^X}{h_t^X} = B \left( A \beta \frac{1 - \alpha + \phi}{\omega + \Omega} \right) \left( \frac{\tau^X}{\kappa} \right)^{\alpha \beta}.
\]

Hence the proof. ■

Proof of Proposition 1. The proof of Part 1 relies on the comparison between \((e_t^L, k_{t+1}^L)\) and \((e_t^G, k_{t+1}^G)\) element-wise. To prove \(e_t^G > e_t^L\), we need to show that \(\theta^G > \beta \delta \phi / (1 + \delta + \beta \delta \phi)\) and for \(k_{t+1}^G < k_{t+1}^L\), we have to show that \((1 + \delta) / (1 + \delta + \beta \delta \phi) > (1 - \theta^G)\). Using the equilibrium value of \(\theta^G\), it is straightforward to verify that both the above conditions are equivalent to the condition \(\delta \beta (1 - \alpha) (1 + \delta + \phi \delta) / \Omega > 0\) and which always holds given our specifications.

For the second part, note that the long-run growth rate in the political economy with public education is higher than that in the laissez-faire economy if and only if \((h_{t+1}^G / h_t^G) \geq (h_{t+1}^L / h_t^L) > 1\) holds at the steady state. Since \((h_{t+1}^G / h_t^G) \geq (h_{t+1}^L / h_t^L)\) is continuous in \(\alpha\), to prove part 2, it is enough to show that \((h_{t+1}^G / h_t^G) \geq (h_{t+1}^L / h_t^L) > 1\) holds when \(\alpha = 0\). Given \(\alpha = 0\), some algebra yields \((h_{t+1}^G / h_t^G) \geq (h_{t+1}^L / h_t^L) = \left[ \theta^G (1 + \delta + \beta \delta \phi) / (\beta \delta \phi) \right]^{\beta}\), which is larger than one by directly following the proof of part 1 presented above. Hence the proof. ■

Proof of Proposition 2. Using the equilibrium factor prices, government budget and the expression for \(k_{t+1}^P\), some tedious algebra yields the equilibrium investment in education

\[
e_t^P = \frac{\omega \beta \delta \phi \Omega}{(\omega + \Omega) [\alpha \delta \Omega + \omega (1 + \alpha \delta + \beta \delta \phi)]} A \kappa_t^\alpha h_t^{1-\alpha}.
\]

In addition, we have \(e_t^X = \theta_t^X w_t h_t = \beta \delta (1 - \alpha + \phi) A \kappa_t^\alpha h_t^{1-\alpha} / (\omega + \Omega)\), from which we can get

\[
e_t^X - e_t^P = \frac{\beta \delta A \kappa_t^\alpha h_t^{1-\alpha}}{\omega + \Omega} \frac{\alpha \delta \Omega (1 - \alpha + \phi) + \omega (1 - \alpha) (1 + \alpha \delta)}{\alpha \delta \Omega + \omega (1 + \alpha \delta + \beta \delta \phi)} > 0.
\]

Further, it is straightforward that

\[
\frac{k_{t+1}^P}{k_{t+1}^X} = \frac{\omega + \alpha \delta \Omega / (1 + \alpha \delta)}{\omega (1 + \alpha \delta + \beta \delta \phi) / \Omega + \alpha \delta}.
\]

Since it can be verified that \((1 + \alpha \delta + \beta \delta \phi) / \Omega < 1\) and \(\Omega / (1 + \alpha \delta) > 1\), obviously we have \(k_{t+1}^P / k_{t+1}^X > 1\). Hence the proof. ■
Proof of Proposition 4. Firstly it can be shown that

\[ \frac{k_{t+1}^L}{k_{t+1}^P} = \frac{1 - \alpha}{\alpha (1 + \delta + \beta \delta \phi)} \left[ \alpha \delta + \frac{\omega (1 + \alpha \delta + \beta \delta \phi)}{\Omega} \right]. \]

and there exists an unique threshold value \( \tilde{\omega} = \alpha \Omega / (1 - \alpha) \) such that \( k_{t+1}^L \geq k_{t+1}^P \) if and only if \( \omega \geq \tilde{\omega} \). Hence the proof.

Next we compare the levels of investment in education. It is straightforward to check that

\[ \frac{e_t^L}{e_t^P} = \frac{1 - \alpha}{1 + \delta + \beta \delta \phi} \left[ 1 + 2 \alpha \delta + \beta \delta \phi + \frac{\omega^2 (1 + \alpha \delta + \beta \delta \phi) + \alpha \delta \Omega^2}{\omega \Omega} \right], \]

from which we know the comparison of \( e_t^L \) with \( e_t^P \) depends on the value of \( \omega \). Taking derivative of \( e_t^L/e_t^P \) with respect to \( \omega \) yields

\[ \frac{\partial}{\partial \omega} \left( \frac{e_t^L}{e_t^P} \right) = \frac{1 - \alpha}{1 + \delta + \beta \delta \phi} \left( \frac{1 + \alpha \delta + \beta \delta \phi}{\Omega} - \frac{\alpha \delta \Omega}{\omega^2} \right). \]

Evidently \( e_t^L/e_t^P \) is convex in \( \omega \) for all \( t \), along with \( e_t^L/e_t^P \to \infty \) as \( \omega \to 0 \) or \( \omega \to \infty \). Hence either \( e_t^L/e_t^P = 1 \) has two roots or \( e_t^L/e_t^P > 1 \) for all \( \omega \). It can be checked that \( \tilde{\omega} \) is one of the solutions to \( e_t^L/e_t^P = 1 \). Therefore there must exist another root of \( e_t^L/e_t^P = 1 \). Denote \( \tilde{\omega} \) as the other root for \( e_t^L/e_t^P = 1 \). By solving \( e_t^L/e_t^P = 1 \), we have

\[ \tilde{\omega} = \frac{\delta (1 - \alpha) \Omega}{1 + \alpha \delta + \beta \delta \phi}. \]

Further, it can be checked that

\[ \left. \frac{\partial}{\partial \omega} \left( \frac{e_t^L}{e_t^P} \right) \right|_{\omega = \tilde{\omega}} = \frac{\alpha + \alpha \beta \delta \phi + 2 \alpha \delta - \delta}{\alpha}. \]

Thus \( \alpha + \alpha \beta \delta \phi + 2 \alpha \delta - \delta \geq 0 \iff \tilde{\omega} \geq \bar{\omega} \) and since \( e_t^L/e_t^P \) is convex in \( \omega \), we must have \( e_t^L < e_t^P \) for \( \omega \in [\bar{\omega}, \tilde{\omega}] \) or \( [\tilde{\omega}, \bar{\omega}] \) depending on which root is bigger, and clearly \( e_t^L > e_t^P \forall \omega \notin [\bar{\omega}, \tilde{\omega}] \) or \( [\tilde{\omega}, \bar{\omega}] \). Hence the proof.

Proof of Proposition 5. First we have

\[ \frac{k_{t+1}^G}{k_{t+1}^X} = \frac{1 - \alpha}{\alpha (1 + \delta)} \left[ \alpha \delta + \frac{\omega (1 + \alpha \delta)}{\Omega} \right], \]

\[ \frac{e_t^G}{e_t^X} = \left( 1 + \frac{\omega}{\Omega} \right) (1 - \alpha). \]

Then it can be easily checked that \( k_{t+1}^G \geq k_{t+1}^X \) and \( e_t^G \geq e_t^X \) if and only if \( \omega \geq \tilde{\omega} \). Hence the proof.
Proof of Proposition 6. It can be verified that

\[
\frac{k_{t+1}^L}{k_{t+1}^X} = \frac{1 - \alpha}{\alpha (1 + \delta + \beta \delta \phi)} \left( \omega + \frac{\alpha \delta \Omega}{1 + \alpha \delta} \right)
\]

which is monotonically increasing with respect to the parameter \( \omega \). Hence there exists a threshold value

\[
\bar{\omega}_1 = \alpha \frac{(1 + \alpha \delta + \beta \delta \phi) (1 + \alpha \delta) - \delta \beta \delta (1 - \alpha + \phi) (1 - \alpha)}{(1 - \alpha) (1 + \alpha \delta)}
\]

such that \( k_{t+1}^L \leq k_{t+1}^X \) if and only if \( \omega \leq \bar{\omega}_1 \).

Similarly, we have

\[
\frac{e_t^L}{e_t^X} = \frac{\phi (1 - \alpha)}{1 + \delta + \beta \delta \phi} \left( \beta \delta + \frac{1 + \alpha \delta + \omega}{1 - \alpha + \phi} \right).
\]

Evidently there exists a threshold value of \( \bar{\omega}_2 \), particularly,

\[
\bar{\omega}_2 = \frac{(1 + \delta + \alpha \phi \beta \delta) (1 - \alpha + \phi) - \phi (1 - \alpha) (1 + \alpha \delta)}{\phi (1 - \alpha)}
\]

such that \( e_t^L \leq e_t^X \) if and only if \( \omega \leq \bar{\omega}_2 \). With some tedious algebra, it can be shown that \( \bar{\omega}_2 > \bar{\omega}_1 \) is equivalent to

\[
1 + \delta + \delta \phi + \alpha \phi \beta \delta + \frac{\alpha \phi \beta \delta^2 (1 - \alpha + \phi)}{1 + \alpha \delta} > 0
\]

and which is obviously true. Hence the proof. ■